Ph.D. Dissertation

Insurance, Social Security, and Economy

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Sungkyunkwan University
Department of Mathematics

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Abstract

Insurance, Social Security, and Economy

Unlike traditional economic models, this study incorporates insurance into the OLG model and analyzes the impact of insurance on economic agents' life cycle behavior and the economy. This study is based on Gertler's (1999) model and focuses primarily on three issues: rederivation, modification, and insurance model development. The contributions and results of this study are as follows.

First, this study provides a clear and detailed derivation of decision problems for economic agents, including variable labor supply, in Gertler's OLG model. Then, based on the Korean economic data, the model is applied to the Korean economy by calibration.

Second, this study modifies Gertler's model with refinement for two factors in worker−decision problems: the risk adjustment factor and valuation factors. In Gertler's model, valuations in non−financial assets are inconsistent, and the steady−state value of social security wealth tends to be overestimated due to the ambiguous transition factors in valuation. Therefore, this study ensures consistency in the valuation by modifying these factors and then compares the two models.

Third, this study presents an insurance model incorporating insurance sections into the modified Gertler's OLG model. Insurance demand is determined by the agents' decision problems, and individuals' insurance purchases directly increase their utility. However, there is a limitation that losses that are not fully compensated by partial private insurance are not reflected in utility. Thus, social insurance is introduced to compensate for losses that are not fully compensated by private insurance, assuming that the proportion of total losses to GDP is constant. That is, it establishes a system in which the private and public sectors share the role of providing compensation for losses. The insurance model has the advantage of being able to recognize losses from particular financial events that existing economic models do not recognize. It is also possible to analyze the microeconomic and macroeconomic effects of insurance.

Finally, this study proposes a life insurance model, an extension of the insurance model. The life insurance model is theoretically more sophisticated than the insurance model in that it reflects a more realistic population structure, asset allocation, and diversification of insurance types. By modifying the basic assumptions about population composition and transition probability, workers' conditions are diversified, and dependents are allowed to receive inheritance and death benefits when workers die.

Keywords : overlapping generation model, insurance, social security, valuation, steady state

Chapter 1. Introduction

As an alternative to the well-known Blanchard's (1985) paper, Gertler develops a highly tractable Overlapping Generation (OLG) model that captures life cycle behavior. Gertler's model is useful for quantitative policy analysis by allowing realistic working or retirement periods, and life cycle setup can be easily used to integrate into other existing growth models.

Despite the advantages of this model, there is little relevant theoretical study of Gertler's model, and it is not widely used as a tool for economic analysis. Thus, this work carries out in-depth theoretical studies of Gertler's model, corrects errors in the model, and presents a new type of OLG model that incorporates the insurance sector for more realistic economic analysis. In other words, to better apply Gertler's model, this paper mainly focuses on three problems: rederivation, modification, and development of insurance models within Gertler's OLG framework.

The baseline formulation of Gertler's (1999) paper mainly assumes an inelastic labor supply, and the theoretical development of economic agents' decision problems, including elastic labor supply, is relatively ambiguous. Therefore, this study provides a clear and detailed derivation of economic agents' decision problems under an elastic labor supply. Then, it is applied to the Korean economy by calibration based on Korean economic data. It also analyzes the impact of social security policy and current social issues such as aging and retirement age extension on the Korean economy in steady−state and discusses the implications of the analysis.

Gertler's model also needs to be modified the two factors for valuation in worker's decision problems. First, the risk adjustment factor should be rederived by applying the Envelope Theorem. Second, the inconsistency in valuation factors in non−financial assets should be improved in the transition from employment to retirement. Therefore, this study modifies Gertler's model by rigorously deriving the above two factors in valuations and compares the two models. The key differences between Gertler's model and the modified model are as follows: first, the definition and role of the risk adjustment factor in the two models are different. Second, the worker's non−financial assets' valuations are consistent in our proposed model but not in Gertler's model. In particular, the valuation of social security wealth in Gertler's model contains an ambiguous transition factor, and it overestimates the steady−state value of social security wealth, compared with the result of our proposed model.

Finally, an insurance model is developed by incorporating the insurance sector into the modified Gertler's OLG model. Despite the growth of the insurance industry, most economic models do not include the insurance sector in the model and overlook the impact of insurance on the economy, such as risk allocation, financial loss compensation, asset transfers, and capital accumulation. For the most part, previous studies on the relationship between insurance and the economy tended to focus on empirical research rather than theoretical research. Thus, the purpose of this study is to provide a novel theoretical framework for considering insurance to analyze the microeconomic and macroeconomic effects of insurance.

The two types of insurance models sequentially are presented. The first is an insurance model that does not include life insurance. Insurance demand is determined by the agents' decision problems, and individuals' insurance purchases directly increase their utility. However, there is a limitation that losses that are not fully covered by partial private insurance are not reflected in utility. Thus, social insurance is introduced to compensate for losses that are not fully covered by private insurance, assuming that the proportion of total losses to GDP is constant. It establishes a system in which the private and public sectors share the role of providing compensation for losses. The insurance model has the advantage of being able to recognize losses from particular financial events that existing economic models do not recognize. It is also possible to analyze the economic effects of changes in the relative proportion of indemnification to total loss by private insurance or by social insurance.

The second is a life insurance model, an extension of the insurance model previously introduced. The life insurance model is theoretically more sophisticated than the insurance model in that it reflects a more realistic population structure, asset allocation, and diversification of insurance types. The life insurance model gives dependents besides workers and retirees the role of economic agents. As the possibility of workers' death is imposed, workers' conditions are diversified, and risk adjustment factors and valuation factors used in decision making are changed. It allows a dependent on receiving inheritance and death benefits from life insurance in the event of a worker's death. Therefore, it is possible to analyze the impact of life insurance purchases on the transfer of assets between workers and dependents. It is also possible to analyze the impacts of increasing the workers' life−insurance purchases on economic agents' life−cycle behavior and the economy.

The structure of this thesis is as follows: Chapter 2 considers the relevant precedent literature of OLG models such as Diamond (1965), Yaari (1965), Blanchard (1985), and describes the characteristics of Gertler's theory, and then provides a rigorous derivation of economic agents' decision problems including elastic labor supply in Gertler's OLG model. Chapter 3 applies the model to the Korean economy by calibration based on Korean economic data. Chapter 4 suggests a modified version of Gertler's OLG model with refinement for two factors of worker's decision problems. Chapter 5 deals with a new type of OLG model in which insurance is incorporated and studies the role of private and social insurance. Chapter 6 provides a life insurance model by modifying population structure and transition probability. Chapter 7 describes the insurance model's applications and examines the effects of increasing the proportion of life insurance on the economic agents and economy. Chapter 8 summarizes the implications and acknowledges the limitations of this study.

Chapter 2. Rederivation of Gertler's OLG model

1. Diamond, Yaari, and Blanchard

Diamond (1965) proposes a basic discrete-time overlapping generations growth model based on Samuelson (1958). He assumes two periods of life, such as work and retirement, and assumes that there are two generations of the young and the old alive at any point in time. Individuals consume a fraction of their income in the first period and save the rest to cover their consumption when they are older. The young's assets at the end of the first period are the source of capital used for aggregate production in the second period, and the old in the first period own the entire capital stock and consume all of it, so there are no savings by the old. Diamond assumes a perfectly competitive labor and capital market and constant return to scale technology in the production function. A simple population and age structure are used in this paper to avoid the need for aggregation. Using this model, Diamond examines long-term competitive equilibrium in a growth model and explores the effect on government debt equilibrium.

Yaari (1965) establishes a situation where consumers can solve life's uncertainty by purchasing or selling actuarial notes. He thinks buying an actuarial note is the same concept as buying an annuity. Consumers who buy an actuarial note receive higher returns than market rates from insurance companies during their lifetime, but their assets are all attributed to insurance companies when they die. Yaari assumes an individual's finite life, which enables simple aggregation of consumption. He investigates the optimal consumption plan and the optimal saving plan by dividing them into four cases, depending on whether actuarial notes are available and utility function assumptions. The utility function assumption is either a Fisher utility function with constraint or a Marshall utility function with no constraint.

Blanchard (1985) develops a continuous-time OLG model. He derives a manageable and straightforward form of aggregate functions, assuming that the probability of death is constant and independent of the consumer's age. This simple form of aggregate function makes it relatively easy to analyze the steady-state of many macroeconomic issues, the dynamic effects of fiscal policy and social insurance. Therefore, the aggregation method of this paper has since been widely used in other studies. Blanchard explains the role of finite horizons and declining labor income in determining interest rates, respectively. In addition, he clarifies the role and the effects of fiscal policies such as government debt, spending, and deficits in determining interest rates. He also rigorously describes the effect of intertemporal tax reallocation, assuming a finite life of the agent. Blanchard's OLG model is tractable but has the limitation that life cycle behavior is not captured in the model.

2. The critical characteristics of Gertler's theory

Gertler (1999) develops an OLG model that is analytically tractable and can capture life−cycle behavior. Gertler assumes an individual's life is finite, and their lifetime is distinguished by the working period and the retirement period. Therefore, there are two types of economic agents in this economy: workers and retirees. Gertler makes three kinds of assumptions about population dynamics, actuarial notes, and preferences to derive a tractable aggregate consumption function.

1) Population dynamics

Constant transition probabilities are imposed per period for workers moving into retirement and retirees moving into death, and the transition probabilities are selectable to establish realistic average periods of life, work, and retirement. Each individual is born a worker and is transferred to maintaining a worker's status or to retirement in the next period by transition probabilities, ω or $1-\omega$, respectively. The probability of survival or death for a worker does not need to be considered because it is assumed that an individual's death begins once he retires. A retiree survives with a probability of γ or is transferred to a state of death with a probability of $1-\gamma$. The remaining expected working period of a worker is $1/(1-\omega)$, and the remaining life expectancy of a retiree is $1/(1-\gamma)$, respectively, on the assumption that the transition probability is independent of employment tenure or age.

Let N_t denote the number of workers in period t and is assumed to grow at the constant rate n. Workers in period t+1 consist of workers who have maintained their status since time t and new workers born in t+1. Thus, the number of new workers at t+1 is $(1-\omega+n)N_t$.

$$N_{t+1} = (1+n)N_t = \omega N_t + (1-\omega+n)N_t . \tag{2.2.1}$$

A ratio of retirees to workers ψ can be derived by using the following equations (2.2.2), and in the stationary equilibrium, the ratio is fixed. Then the number of retirees at time t is ψN_t, and the number of retirees grows at rate n, like workers' growth rate.

$$\sum_{s=1}^{\infty} \gamma^{s-1}(1-\omega)N_{t-s} = \sum_{s=1}^{\infty} \gamma^{s-1}(1-\omega)\tfrac{N_t}{(1+n)^s} = \tfrac{(1-\omega)N_t}{1+n}\sum_{s=1}^{\infty}\left(\tfrac{\gamma}{1+n}\right)^{s-1} = \tfrac{(1-\omega)N_t}{1+n-\gamma} , \tag{2.2.2}$$

$$\psi = \tfrac{1-\omega}{1+n-\gamma} . \tag{2.2.3}$$

Table 2-1. Transition probabilities

	Worker	Retiree
Survival rate	N/A	γ
Probability of remaining in the labor force	ω	γ
Probability of transition from labor force	$1-\omega$	$1-\gamma$

Notes: Each transition probability is greater than zero and less than one.

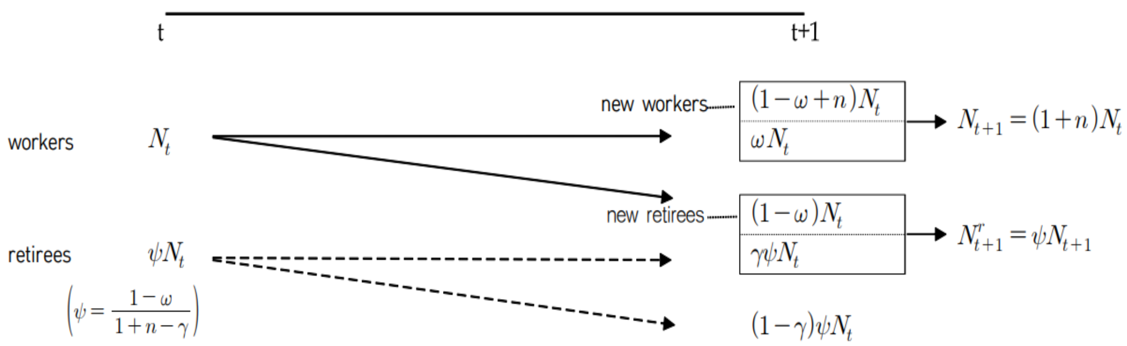

Figure 2−1. Population dynamics by transition probability

2) Actuarial note

The notion of an actuarial note was initially introduced by Yaari (1965) and then applied by Blanchard (1985), Gertler (1999) to their model. An actuarial note is intended to eliminate the impact of an uncertain life span. When the person who purchases the actuarial note dies, the person's property is attributed to the insurance company and distributed to the rest of alive. Therefore, alive persons can get higher returns than the normal returns on the assets they entrust. An actuarial note in this model is limited to retirees, and under the arrangement, each retiree buys an actuarial note. For a retiree, fraction γ of those that survive to the next period receives all returns, while fraction $1-\gamma$ who die receive nothing. Each surviving retiree receives a return in proportion to his initial wealth. Thus, for example, if R is the gross return on assets, a surviving retiree's gross return on wealth is R/γ.

3) Preferences

Individuals are assumed to be preferred to separate risk aversion from intertemporal substitution to address an income risk. The assumption of risk neutrality is reasonable in that it mitigates the impact of income variation that occurs on the assumption of the constant probability of transition in the model. Thus, we adopt a non−expected utility function proposed by Farmer (1990) that limits individuals to have risk neutrality with income risk but to be arbitrary for intertemporal elasticity of substitution. V_t^i is an individual's value function, where the superscript $i = w, r$ denotes a worker (w) or a retiree (r) respectively. An individual derives utility from consumption C_t^i, leisure $1 - l_t^i$, where l_t^i is the fraction of working time at time t by an agent. β^i is a subjective discount factor and γ is a retiree's survival rate for each period. Then, the value function of an individual is given by:

$$V_t^i = \left\{ [(C_t^i)^{\upsilon}(1 - l_t^i)^{1-\upsilon}]^{\rho} + \beta^i \left\{ E_t(V_{t+1} \mid i) \right\}^{\rho} \right\}^{1/\rho}, \qquad (2.2.4)$$

$$\text{with} \qquad \beta^w = \beta, \quad \beta^r = \beta\gamma,$$

where $E_t(V_{t+1} \mid i)$ is the expected value of next periods' value function, conditional on the person being type i at t and being alive at t+1.

$$E_t(V_{t+1} \mid w) = \omega V_{t+1}^w + (1 - \omega) V_{t+1}^r, \qquad (2.2.5)$$

$$E_t(V_{t+1}^r \mid r) = V_{t+1}^r.$$

The retiree's effective intertemporal subjective discount factor is $\beta\gamma$ since the survival rate until the next period is γ. In other words, the expected value function is considered in the next period on the assumption that the retiree is alive in the next period. Meanwhile, the curvature parameter ρ introduces a smooth trade-off for individuals between intertemporal consumption. The intertemporal elasticity of substitution, $\sigma = 1/(1-\rho)$, is finite and choice of σ is flexible under this preference structure.

Gertler's model is meaningful because it embeds life-cycle behavior within a dynamic general equilibrium economy. It also makes many policy experiments and extensions possible through its tractability. In Gertler's model, there are two endogenous state variables: capital stock and the share of financial assets. The change in the economic agents' financial assets share due to demographic changes or policy changes can explain the transfer of wealth between workers and retirees.

3. Rederivation of Gertler's OLG model including elastic labor

The baseline formulation of Gertler's paper assumes inelastic labor supply, so the theoretical development of economic agents' decision problems, including elastic labor supply, is relatively ambiguous. Therefore, we provide a clear and detailed derivation of economic agents' decision problems, including labor supply in Gertler's OLG model.

1) Individual decision

Each individual has a labor−leisure choice for one unit of time per period under an elastic labor supply. Since wage levels depend on labor productivity, it is assumed that a retiree receives relatively low wages than a worker. Let $\xi \in (0,1)$ be the productivity of labor supplied by a retiree relative to a worker, and then the wage per unit of time is as follows: $W^w = W$, $W^r = \xi W$. Furthermore, there is a government that implements fiscal and social security policies. Social security benefits are paid only to retirees, and taxes are levied only on workers to finance the government's policies.

(1) Retiree−decision problem

Retirees choose consumption C_t^r , leisure $1-l_t^r$ that maximize their value function

$$V_t^r = \left\{ [(C_t^r)^{\upsilon}(1-l_t^r)^{1-\upsilon}]^{\rho} + \beta\gamma(V_{t+1}^r)^{\rho} \right\}^{1/\rho} , \qquad (2.3.1)$$

subject to budget constraint

$$A_{t+1}^r = \tfrac{R_t}{\gamma} A_t^r + W_t^r l_t^r + E_t - C_t^r . \qquad (2.3.2)$$

Financial assets A_t^r at the beginning of the period from t and t+1 evolve according to Eq. (2.3.2). R_t / γ is the gross return applied to retirees buying an actuarial note and surviving up to t+1, $W_t^r l_t^r$ is labor income, E_t is social security benefits from the government, C_t^r is consumption at the end of the period t.

Retirees consume out of current assets, including financial and non−financial assets. Non−financial assets include human wealth H_t^r that is the present value of the future labor income and social security wealth S_t^r that is the present value of the future social security benefits, including time t−point. The retiree's non−financial assets can be evaluated as

$$H_t^r = W_t^r l_t^r + \tfrac{\gamma}{R_{t+1}} H_{t+1}^r, \tag{2.3.3}$$

$$S_t^r = E_t + \tfrac{\gamma}{R_{t+1}} S_{t+1}^r. \tag{2.3.4}$$

The first−order necessary condition for labor is given by (Appendix (A.1.7))

$$1 - l_t^r = \tfrac{1-\upsilon}{\upsilon} C_t^r / W_t^r, \tag{2.3.5}$$

and the consumption Euler equation for the retiree yields: (Appendix (A.1.17))

$$C_{t+1}^r = \left[\left(\tfrac{W_t^r}{W_{t+1}^r}\right)^{(1-\upsilon)\rho} R_{t+1}\beta \right]^{\sigma} C_t^r. \tag{2.3.6}$$

Let us guess a form of consumption function as follows:

$$C_t^r = \varepsilon_t \pi_t \left(\tfrac{R_t}{\gamma} A_t^r + H_t^r + S_t^r\right), \tag{2.3.7}$$

where $\varepsilon_t \pi_t$ is the retiree's propensity to consume out of wealth. $\varepsilon_t \pi_t$ is derived by (Appendix (A.1.23))

$$\varepsilon_t \pi_t = 1 - \left[\left(\tfrac{W_t^r}{W_{t+1}^r}\right)^{1-\upsilon} R_{t+1}\right]^{\sigma-1} \beta^{\sigma} \gamma \tfrac{\varepsilon_t \pi_t}{\varepsilon_{t+1}\pi_{t+1}}. \tag{2.3.8}$$

ε_t is the ratio of two propensity to consume, which can be interpreted as the retiree's consumption elasticity.

(2) Worker−decision problems

Workers choose consumption C_t^w, leisure $1-l_t^w$ that maximize their value function

$$V_t^w = \left\{ [(C_t^w)^\upsilon (1-l_t^w)^{1-\upsilon}]^\rho + \beta[\omega V_{t+1}^w + (1-\omega)V_{t+1}^r]^\rho \right\}^{1/\rho}, \tag{2.3.9}$$

subject to budget constraint

$$A_{t+1}^w = R_t A_t^w + W_t\, l_t^w - C_t^w. \tag{2.3.10}$$

Financial assets A_t^w at the beginning of the period from t and t+1 evolve according to Eq. (2.3.10). R_t is the gross return, $W_t\, l_t^w$ is after−tax labor income, C_t^w is consumption at the end of the period. Workers do not receive social security benefits during the working period, but they will receive them from then on once they retire.

As with retirees, workers spend on current assets, including financial and non−financial assets. On the other hand, the valuation of a worker's non−financial assets should take into account future cash flows in accordance with two possibilities of maintaining worker status and retiring by transition probabilities. Therefore, the worker's non−financial assets, human wealth H_t^w and social security wealth S_t^w, are evaluated by

$$H_t^w = W_t l_t^w + \tfrac{1}{R_{t+1}\Omega_{t+1}} \omega H_{t+1}^w + \tfrac{1}{R_{t+1}\Omega_{t+1}} (1-\omega) H_{t+1}^r, \tag{2.3.11}$$

$$S_t^w = \tfrac{1}{R_{t+1}\Omega_{t+1}} \omega S_{t+1}^w + \tfrac{1}{R_{t+1}\Omega_{t+1}} (1-\omega)\varepsilon_{t+1} \tfrac{1}{\psi N_t} S_{t+1}^r, \tag{2.3.12}$$

using two discounting factors depending on the two cases of work and retirement. In Gertler (1999), $\frac{1}{\psi N_t} S^r_{t+1}$ is defined as the social security wealth per beneficiary at t+1, and ε_{t+1} is interpreted as the value that workers able to consume today from social security wealth to be received after retirement.

In the worker-decision problems, the leisure equations and consumption Euler equation for the worker are derived as follows: (Appendix (A.2.7) and (A.2.8))

$$1-l_t^w = \tfrac{1-\upsilon}{\upsilon} C_t^w / W_t \tag{2.3.13}$$

$$\omega C^w_{t+1} + (1-\omega)\chi(\varepsilon_{t+1})^{\frac{\sigma}{1-\sigma}} C^r_{t+1} = [(\tfrac{W_t}{W_{t+1}})^{1-\upsilon}]^{\sigma-1}(R_{t+1}\Omega_{t+1}\beta)^{\sigma} C_t^w . \tag{2.3.14}$$

In Eq. (2.3.12), an adjusting discounting factor Ω_{t+1} is defined as follows:(Appendix (A.2.17))

$$\Omega_{t+1} = \omega + (1-\omega)(\varepsilon_{t+1})^{\frac{1}{1-\sigma}}\chi , \tag{2.3.15}$$

where $\chi = (\frac{1}{\xi})^{1-\upsilon}$, $\xi = \frac{W^r_{t+1}}{W_{t+1}}$.

χ reflects the relative wages of workers and retirees. Assuming that every worker consumes a fraction π_t of his wealth, the form of consumption functions similar to those of a retiree can be inferred as follows:

$$C_t^w = \pi_t (R_t A_t^w + H_t^w + S_t^w) , \tag{2.3.16}$$

where $\xi = \frac{W^r_{t+1}}{W_{t+1}}$

$$\pi_t = 1 - [(\tfrac{W_t}{W_{t+1}})^{1-\upsilon} R_{t+1}\Omega_{t+1}]^{\sigma-1}\beta^{\sigma}\tfrac{\pi_t}{\pi_{t+1}} . \tag{2.3.17}$$

2) Aggregate decision

(1) Aggregate consumption and the transfer of wealth

Since individuals within a group have the same propensity to consume out of wealth, one can sum (2.3.7) and (2.3.16) across individual retirees/workers to derive the aggregate consumption function by groups as follows:

$$C_t^{r\bullet} = \varepsilon_t \pi_t (R_t A_t^{r\bullet} + H_t^{r\bullet} + S_t^{r\bullet}) = \varepsilon_t \pi_t (R_t \lambda_t^r A_t^{r\bullet} + H_t^{r\bullet} + S_t^{r\bullet}), \qquad (2.3.18)$$

$$C_t^{w\bullet} = \pi_t (R_t A_t^{w\bullet} + H_t^{w\bullet} + S_t^{w\bullet}) = \pi_t [R_t (1-\lambda_t^r) A_t^{\bullet} + H_t^{w\bullet} + S_t^{w\bullet}]. \qquad (2.3.19)$$

In (2.3.18), (2.3.19), $\lambda_t^r \equiv A_t^{r\bullet} / A_t^{\bullet}$ and $\lambda_t^w \equiv (1-\lambda_t^r) \equiv A_t^{w\bullet} / A_t^{\bullet}$ denote the share of assets held by retirees and workers, respectively. The gross return on retirees' financial assets in (2.3.18) is R_t because the surviving retirees are the fraction γ of the total number of retirees.

Then, (2.3.18) and (2.3.19) are combined to obtain the following aggregate consumption function:

$$C_t^{\bullet} = \pi_t [\{1 + (\varepsilon_t - 1)\lambda_t^r\} R_t A_t^{\bullet} + H_t^{w\bullet} + S_t^{w\bullet} + \varepsilon_t (H_t^{r\bullet} + S_t^{r\bullet})]. \qquad (2.3.20)$$

The aggregate human wealth and social security wealth equations of retirees are given by

$$H_t^{r\bullet} = W_t^r L_t^r + \tfrac{\gamma}{(1+n)R_{t+1}} H_{t+1}^{r\bullet}, \qquad (2.3.21)$$

$$S_t^{r\bullet} = E_t^{r\bullet} + \tfrac{\gamma}{(1+n)R_{t+1}} S_{t+1}^{r\bullet}, \qquad (2.3.22)$$

which are derived from summing (2.3.3) and (2.3.4) over individual retirees.

Since the workforce increases by (1+n) over the period, it is necessary to adjust the value of future non−financial assets at t+1 to the t−point value using the dividing factor (1+n).

For workers, the aggregate human wealth and social security wealth equations are given by

$$H_t^{w\bullet} = W_t\, L_t^w - T_t + \tfrac{1}{(1+n)R_{t+1}\Omega_{t+1}}\omega H_{t+1}^{w\bullet} + \tfrac{1}{(1+n)R_{t+1}\Omega_{t+1}}(1-\omega)H_{t+1}^{r\bullet} \tag{2.3.23}$$

$$S_t^{w\bullet} = \tfrac{1}{(1+n)R_{t+1}\Omega_{t+1}}\omega S_{t+1}^{w\bullet} + \tfrac{1}{R_{t+1}\Omega_{t+1}}(1-\omega)\tfrac{\varepsilon_{t+1}}{(1+n)\psi}S_{t+1}^{r\bullet}. \tag{2.3.24}$$

Equations (2.3.23) and (2.3.24) are respectively derived from summing (2.3.11), (2.3.12) over individual workers. In (2.3.23), the total labor income of all workers $W_t\, L_t^w$ is pre−tax income; thus, the total tax on labor income for all workers T_t should be deducted. As moving between groups by transition probability, the distribution of financial wealth evolves and a change in the distribution of financial wealth will influence each group's total consumption demand.

Now I derive the equation of the share of financial assets by groups λ_t^i, and examine the evolution of the distribution of wealth. The total financial assets held by workers at t+1, equal assets accumulated by workers at t for t+1 times the probability of remaining in the workforce in the next period ω, as follows:

$$(1-\lambda_{t+1}^r)A_{t+1}^\bullet = \omega[(1-\lambda_t^r)R_t A_t^\bullet + W_t L_t^w - T_t - C_t^{w\bullet}], \tag{2.3.25}$$

which can be rewritten as

$$R_t \lambda_t^w A_t^\bullet + W_t L_t^w - T_t - C_t^{w\bullet} = \tfrac{(1-\lambda_{t+1}^r)}{\omega}A_{t+1}^\bullet. \tag{2.3.26}$$

In (2.3.25), the square bracket means workers' assets before distribution by movement from workers to retirees by transition probability and workers' total savings at the end of time t.

Total financial assets owned by retirees at the beginning of period t+1 depend on the savings of current retirees at t and the savings of workers retiring at t+1. In other words, the total assets held by retirees at t+1, equal to the sum of assets accumulated by existing retirees and assets carried by new retirees. Thus, the equation for the total financial assets held by retirees at t+1 is derived as follows:

$$\lambda^r_{t+1}A^\bullet_{t+1} = R_t\lambda^r_t A^\bullet_t + W^r_t L^r_t + E^\bullet_t - C^{r\bullet}_t + (1-\omega)\times(R_t\lambda^w_t A^\bullet_t + W_t L^w_t - T_t - C^{w\bullet}_t)\,. \qquad (2.3.27)$$

By using (2.3.26), (2.3.27) can be rewritten as

$$\lambda^r_{t+1}A^\bullet_{t+1} = R_t\lambda^r_t A^\bullet_t + W^r_t L^r_t + E^\bullet_t - C^{r\bullet}_t + (1-\omega)\tfrac{(1-\lambda^r_{t+1})}{\omega}A^\bullet_{t+1}\,. \qquad (2.3.28)$$

Summarize the equation for λ^r_{t+1} after putting (2.3.18) into (2.3.28), then the share of wealth held by retirees evolves according to

$$\lambda^r_{t+1} = \omega(1-\varepsilon_t\pi_t)R_t\lambda^r_t\tfrac{A^\bullet_t}{A^\bullet_{t+1}} + \omega[W^r_t L^r_t + E^\bullet_t - \varepsilon_t\pi_t(H^{r\bullet}_t + S^{r\bullet}_t)]\tfrac{1}{A^\bullet_{t+1}} + (1-\omega)\,. \qquad (2.3.29)$$

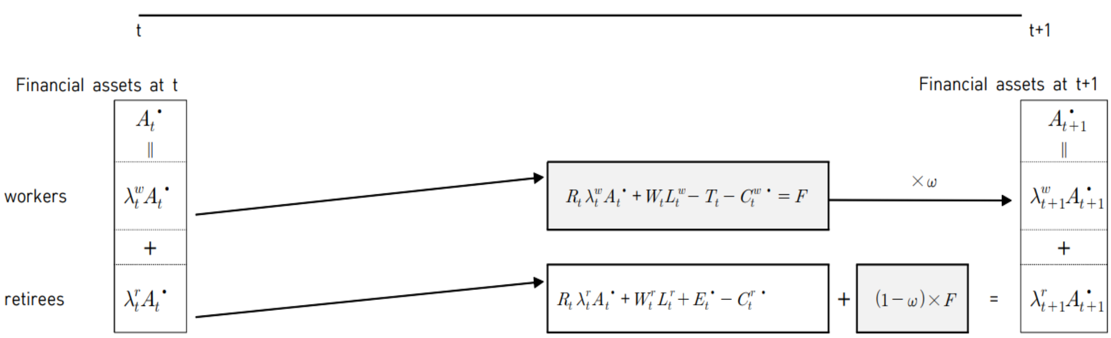

Figure 2-2. Change in the share of assets by economic agents from t to t+1

(2) Production, social security and government

Under a closed and a competitive economy, the total production Y_t is given by a Cobb−Douglas production function that is a standard form for production as follows:

$$Y_t = (X_t N_t)^{\alpha} K_t^{1-\alpha}. \tag{2.3.30}$$

There are two inputs to the production function: labor N and capital K. Capital depreciates at the rate δ, and the parameter α and $1-\alpha$ are the output elasticities of labor and capital, respectively. Technology X is labor−augmenting and grows exogenously by x and X_t^{α} is the total factor productivity.

$$X_{t+1} = (1+x) X_t \tag{2.3.31}$$

On the assumption that both workers and retirees supply labor elastically, the maximum workforce available and the workforce supplied do not always coincide. Thus, total output is now given by

$$Y_t = (X_t L_t)^{\alpha} K_t^{1-\alpha}, \tag{2.3.32}$$

$$\text{where} \qquad L_t = L_t^w + \xi L_t^r. \tag{2.3.33}$$

Aggregate labor supply equations of workers and retirees obtained by merely summing individual labor supply equations (2.3.5), (2.3.13) are given by

$$L_t^w = N_t - \tfrac{(1-\upsilon)/\upsilon}{W_t} C_t^w, \tag{2.3.34}$$

$$L_t^r = N_t \psi - \tfrac{(1-\upsilon)/\upsilon}{W_t} C_t^r. \tag{2.3.35}$$

By the profit−maximizing, Equation (2.3.32) implies

$$W_t = \alpha Y_t / L_t, \tag{2.3.36}$$

$$R_t = (1-\alpha)Y_t / K_t + (1-\delta). \tag{2.3.37}$$

On the other hand, there is a government that implements fiscal and social security policies. Each period, the government consumes G_t and pays retirees a total of social security benefits E_t. To finance the expenditure, the government issues one−period government bonds B_{t+1} and levies a total of tax T_t. Thus, the stock of government debt at the beginning of time t+1 is given by

$$B_{t+1} = R_t B_t + G_t + E_t - T_t. \tag{2.3.38}$$

By iterating equation (2.3.38), the inter−temporal budget constraint is as following by

$$R_t B_t = \sum_{v=0}^{\infty} \frac{T_{t+v}}{\prod_{z=1}^{v} R_{t+z}} - \sum_{v=0}^{\infty} \frac{G_{t+v}}{\prod_{z=1}^{v} R_{t+z}} - \sum_{v=0}^{\infty} \frac{E_{t+v}}{\prod_{z=1}^{v} R_{t+z}}. \tag{2.3.39}$$

The present value of the outstanding debt is equal to the present value of the total government expenditure not covered by taxes for each period. Assume that the government adjusts taxes with other government policies fixed to satisfy equation (2.3.39). The ratio of government consumption to output $\bar{g}_t$, the ratio of social security payments to output $\bar{e}_t$, and the stock of government bonds to output $\bar{b}_t$ are assumed to be fixed as follows:

$$G_t = \bar{g}_t Y_t, \quad E_t = \bar{e}_t Y_t, \quad B_t = \bar{b}_t Y_t. \tag{2.3.40}$$

Financial wealth equals the sum of capital and government debt,

$$A_t = K_t + B_t, \tag{2.3.41}$$

and which are the vehicle for saving, and the capital intensity evolves as

$$K_{t+1} = Y_t - C_t - G_t + (1-\delta)K_t. \tag{2.3.42}$$

(3) Steady−state equations

This section derives the steady−state equations from the previous sections' aggregate functions. The method of deriving the steady−state equations is followed by Gertler (1999), and all steady−state variables are denoted by the normalized certain variables relative to output. For example, the ratio of capital to output is denoted by $k = K/Y$. In the steady−state, all quantity variables grow exogenously at the effective labor force growth rate $(1+x)(1+n)$.

First, let initial values to be R, Ω, L_w/N, L_w/L and by using equation

$$R = (1-\alpha)k^{-1} + (1-\delta). \tag{2.3.43}$$

Then we have initial capital intensity from (2.3.43)

$$k = (1-\alpha)/(R-1+\delta), \tag{2.3.44}$$

which is tentatively determined. Next, the ratio of total tax to output τ can be determined

$$\tau = [R-(1+x)(1+n)]b + g + e. \tag{2.3.45}$$

For each worker and retiree, the following steady−state values of the propensity to consume out of wealth equations are as follows:

$$\pi = 1 - [(\tfrac{1}{1+x})^{1-\upsilon} R\Omega]^{\sigma-1}\beta^{\sigma}, \tag{2.3.46}$$

$$\varepsilon\pi = 1 - [(\tfrac{1}{1+x})^{1-\upsilon} R]^{\sigma-1} \beta^{\sigma} \gamma \,. \tag{2.3.47}$$

Hence, elasticity of retiree's consumption ε is determined.

Workers' and retirees' human wealth can be evaluated as

$$h^{w} = [\alpha \tfrac{L^{w}}{L} - \tau + \tfrac{(1-\omega)(1+x)}{R\Omega} h^{r}] / (1 - \tfrac{\omega(1+x)}{R\Omega}) \,, \tag{2.3.48}$$

$$h^{r} = \alpha \tfrac{L-L^{w}}{L} / (1 - \tfrac{\gamma(1+x)}{R}) \,, \tag{2.3.49}$$

by using recursive human wealth equation, respectively,

$$h^{w} = \alpha \tfrac{L^{w}}{L} - \tau + \tfrac{(1+x)}{R\Omega} \omega h^{w} + \tfrac{(1+x)}{R\Omega} (1-\omega) h^{r} \,, \tag{2.3.50}$$

$$h^{r} = \alpha \tfrac{L-L^{w}}{L} + \tfrac{\gamma(1+x)}{R} h^{r} \,. \tag{2.3.51}$$

Workers' and retirees' social security wealth can be evaluated as

$$s^{w} = \tfrac{(1-\omega)(1+x)}{R\Omega} \tfrac{\varepsilon}{\psi} s^{r} \,/\, (1 - \tfrac{\omega(1+x)}{R\Omega}) \,, \tag{2.3.52}$$

$$s^{r} = e / (1 - \tfrac{\gamma(1+x)}{R}) \,, \tag{2.3.53}$$

by using recursive social security wealth equation, respectively,

$$s^{w} = \tfrac{(1+x)}{R\Omega} \omega s^{w} + \tfrac{(1+x)}{R\Omega} (1-\omega) \tfrac{\varepsilon}{\psi} s^{r} \,, \tag{2.3.54}$$

$$s^{r} = e + \tfrac{\gamma(1+x)}{R} s^{r} \,. \tag{2.3.55}$$

Next, steady-state values of the share of assets for each agent can be expressed and solved in

$$\lambda^{w} = 1 - \lambda^{r} \,, \tag{2.3.56}$$

$$\lambda^{r} = \frac{\omega[\alpha \frac{L-L^{w}}{L} + e - \varepsilon\pi(h^{r} + s^{r})](k+b)^{-1} + (1-\omega)(1+x)(1+n)}{(1+x)(1+n) - \omega R(1-\varepsilon\pi)} \,. \tag{2.3.57}$$

The steady-state version of each aggregate consumption amounts can be

$$c^w = \pi[(1-\lambda^r)R(k+b) + h^w + s^w], \quad (2.3.58)$$

$$c^r = \varepsilon\pi[\lambda^r R(k+b) + h^r + s^r], \quad (2.3.59)$$

and the aggregation equation will be

$$c = \pi\left\{\left[1+(\varepsilon-1)\lambda^r\right]R(k+b) + h^w + s^w + \varepsilon(h^r + s^r)\right\}, \quad (2.3.60)$$

which can be used in calculating capital intensity such that

$$\left[(1+x)(1+n) - 1 + \delta\right]k = 1 - c - g. \quad (2.3.61)$$

Placing (2.3.60) into (2.3.61), then Eq. (2.3.61) is obtained.

$$\left[(1+x)(1+n) - 1 + \delta\right]k = 1 - \pi\left\{[1+(\varepsilon-1)\lambda^r]R(k+b) + h^w + s^w + \varepsilon(h^r + s^r)\right\} - g. \quad (2.3.62)$$

Hence, new recursive values are

$$R = \frac{1-[(1+x)(1+n)-1+\delta]k - \pi[h^w + s^w + \varepsilon(h^r + s^r)] - g}{\pi[1+(\varepsilon-1)\lambda^r](k+b)}, \quad (2.3.62)$$

and the worker's adjusting discount factor and steady-state value related to labor supply are as follows:

$$\Omega = \omega + (1-\omega)(\varepsilon_{t+1})^{\frac{1}{1-\sigma}}\chi, \quad (2.3.63)$$

$$\frac{L^w}{L} = \frac{N}{L} - \frac{(1-\upsilon)/\upsilon}{\alpha}c^w, \quad (2.3.64)$$

$$\frac{L}{N} = (1+\xi\psi)[1 + \frac{(1-\upsilon)/\upsilon}{\alpha}c]^{-1}, \quad (2.3.65)$$

used in minimizing the distance measure of initial and new values of R and Ω, L_w / N, L_w / L.

This chapter was published in Communications for Statistical Applications and Methods on November 30, 2020, and for more information, see Lee and Son (2020).

Chapter 3. Analysis of the Korean economy

1. Initial steady−state values

This chapter applies the model to the Korean economy by calibration based on Korean economic data and analyzes the impact of social security policy and current social issues such as aging and retirement age extension on the Korean economy. Some of the values for the exogenous nonpolicy parameters such as preference parameter for consumption, subjective discount, depreciation rate, intertemporal preference, labor productivity of a retiree are the same as Gertler (1999) choose. Other parameters are set using recent statistics obtainable from official sources of Korea.

The workforce growth rate is set at 0.01 using a 3−year average of recent data obtained from The Statistics Korea and National Accounts. The labor income share ratio is set at 0.628, and the growth rate of technology is set at 0.014 using a 3−year average of recent data obtained from The Bank of Korea. Policy parameter such as the ratio of government debt to output, the ratio of government consumption to output and the ratio of social security payments to output is set at 0.36, 0.15, 0.015, respectively using a 2−year average of data obtained from the government institutions.

Table 3−2 displays the steady−state values of basic variables when applying the exogenous parameter values in Table. 3−1 to the model. In steady−state, the capital−output ratio is 2.10, the capital stock per unit of effective labor is 3.25, and the total return on capital is 1.077. The propensity to consume of a retiree is larger than that of a worker, and the ratio of two propensity to consume out of wealth is more than one. Out of total financial assets, workers' share is about 56.8%, and retirees' share is about 43.2%. The proportion of total tax revenue to GDP is 18.6%, workers' tax burden to pre−tax labor income is about 33% and the ratio of consumption to post−tax income is 0.973.

In this model, only a worker pays income tax, so the proportion of workers' consumption to their post−tax income is considerably high, showing that they have less room to save for capital accumulation. When labor supply represents a fraction of total time endowment, workers supply labor about 50.6% of their total labor force in steady state. Retirees supply labor about 16.6% of their total labor force since they have lower productivity of a unit of labor and lower wages than workers. Moreover, this reflects the fact that retirees have more accumulated financial wealth on average.

The fourth and fifth columns of Table 3−2 shows the steady−state values per capita for each group for easy comparison. The proportion of each group is obtained by using the number of retirees to the number of workers ψ. Of the total population, the proportion of workers is 62.7%, and that of retirees is 37.3%, respectively. For the ratio of human assets to output per capita, per worker is 3.903 and per retiree is 1.290. Since a worker has a more extended working period and higher efficiency per unit of labor than a retiree, the value of human wealth per worker is also higher than that of a retiree. Furthermore, the share of assets per capita is 0.907 for a worker, 1.156 for a retiree. Retirees' financial assets consist of existing assets and transferred assets from retired workers, and the proportion of financial assets per retiree is higher than that of workers.

Table 3-1. Definition and value of parameters

Parameter	Value	Description	Source
n	0.01	workforce growth rate : 3-year (2016-2018) average of the growth rate of working age population aged 20 to 64.	The economically active population survey, Statistics Korea.
ω	0.94	Probability of remaining a worker in the next period : the remaining time in the labor force for a representative worker with an age of 42 is 18 years. ($1/(1-\omega)=18$)	
γ	0.92	Probability of surviving of a retiree to the next period : the remaining life expectancy of a representative retiree with an age of 71 is 12 years. ($1/(1-\gamma)=12$)	
υ	0.4	Preference parameter for consumption	Gertler(1999)
$1-\upsilon$	0.6	Preference parameter for leisure	Gertler(1999)
β	1	Subjective discount rate	Gertler(1999)
ρ	-3	Curvature parameter	Gertler(1999)
σ	0.25	Intertemporal elasticity of substitution	Gertler(1999)
ξ	0.6	Productivity of a unit of labor supplied by a retiree relative to a worker	Gertler(1999)
α	0.628	Labor income share: 3-year	National accounts,

		(2016~2018) average of the labor income share ratio.	the Bank of Korea
δ	0.1	Capital depreciation rate	Gertler(1999)
x	0.014	Growth rate of technology : derived from the 3-year (2016~2018) average of the total factor productivity growth rate by using formula, TFP = x^{α} .	
b	0.36	Government debt to output : 2-year average of the government debt to output.	Electronic national index, Ministry of Economy and Finance
g	0.15	Government consumption to output : 2-year average of government expenditure which is the sum of mandatory (excluding social security payments and interest) and discretionary expenditure.	The FY2017 Accounting Analysis Series Ⅰ, National Assembly Budget Office
e	0.015	Social security payments to output : 2-year average of social security payments to output composed of the old-age pension, disability pension and survivor pension in National Pension Scheme and the basic pension	The FY2017Accounting Analysis Series Ⅰ, National Assembly Budget Office

Table 3-2. Initial steady-state values of basic exogenous variables

Variables	Description	Values	Values per capita
k	Capital stock	2.10	
K / XL	Capital stock per unit of effective labor	3.25	
R	Total return on capital	1.077	
π	Propensity to consume for a worker	0.091	
$\varepsilon\pi$	Propensity to consume for a retiree	0.128	
ε	Elasticity of consumption	1.403	

	of retirees to workers		
Ω	Adjusting discounting factor	1.063	
h^w	Human wealth	2.447	3.903
h^r		0.481	1.290
s^w	Social security wealth	0.140	0.120
s^r		0.231	0.284
$\lambda^w = 1-\lambda^r$	Share of assets	0.568	0.907
λ^r		0.432	1.156
τ	Total tax	0.186	
c^w	Consumption	0.366	0.584
c^r		0.221	0.591
$c = c^w + c^r$		0.587	
$\tau / \alpha \frac{L^w}{L}$	Workers' tax burden	0.330	
L^w / N	Labor supply as a fraction of total time endowment	0.506	
$L^r / N\psi_r$		0.166	

Notes: 1) In the superscript or subscript of variables, w denotes workers, r denotes retirees. 2) Quantitative variables such as capital stock, non-financial assets, tax, consumption are represented as the normalized variables relative to output.

2. Sensitivity analyses

This section carries out various quantitative analyses on the impact of changes in social security changes, aging and the extension of retirement age on the Korean economy.

1) Effects of increasing social security payments

Let us examine the steady-state impact of the ratio of social security payments to GDP by increasing the percentage of expenditure on national pension benefits to GDP by a total of 9%p. The rest of the variables are assumed to be constant, and all the increases in national pension payments come from an increase in benefits per retiree, not an increase in the number of retirees.

The rise in the social security payments-to-GDP ratio causes an asset transfer from workers to retirees because the source of funds to pay retirees more benefits is taxes collected from workers. Figure 3-1 and Figure 3-2 illustrate the changes in the share of financial assets of workers and retirees. Retirees' asset share increases from 43.2% to 49.1%, while workers' asset share decreases from 56.8% to 50.9%, and the share of assets per retiree increases from 1.28 to 1.62 times that of a worker.

As the tax burden on workers is higher than before, workers are required to pay 14.2%p more tax on income, from 33.0% to 47.2%, which is larger than the increments of the proportion of total tax to GDP shown in Figure 3-3 and Table 3-5. When other conditions are constant, raising taxes to cover an increase in the ratio of expenditure on national pension benefits to GDP means increasing the total contribution rate. Therefore, to increase the ratio of expenditure on national pension benefits to GDP by 9%p and not cause a deficit in the balance of payments, the contribution rate of workers should increase by 14.2%p from the current level. (In reality, the total contribution rate is 9% of salaries per annum, with both the employer and employee splitting the 9% contribution equally. However, this model assumes that the total contribution rate is 9% for employees.)

Increasing the ratio of social security payments to GDP negatively affects the retiree's labor supply. Retirees supply far less labor than workers since they have more accumulated financial wealth, and wages are lower than workers. For retirees, the more pension benefits, the fewer incentives to supply the labor force, and thus the labor supply of retirees decreases sharply from 16.6% to 1.9%, as shown in Figure 3−4.

As the social security benefits per retiree received after retirement increase, the social security wealth of both workers and retirees is higher than before, as shown in Figure 3−5. On the other hand, the human wealth of both workers and retirees are lower than before. For workers, an increased tax burden on labor income by increasing the ratio of social security payments to GDP worsens human wealth, although workers' labor supply slightly increases. As previously mentioned, for retirees, the ratio of human wealth to GDP decreases due to the sharp reduction of labor supply.

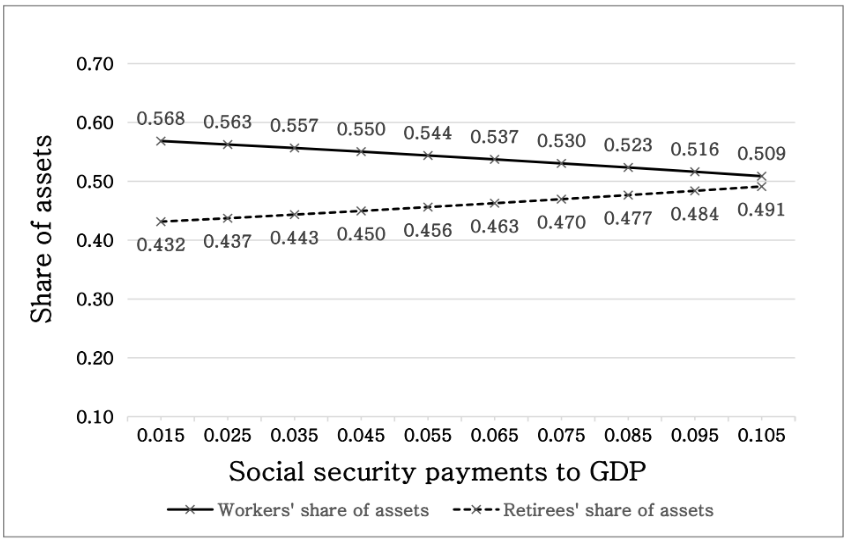

Figure 3−1. Effects of social security payments on the share of assets

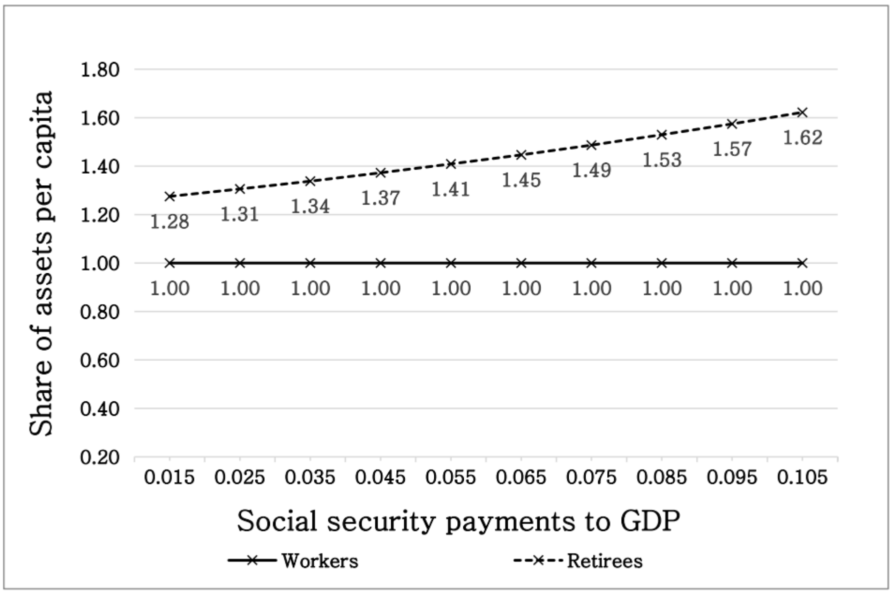

Figure 3-2. Effects of social security payments on the share of assets per capita

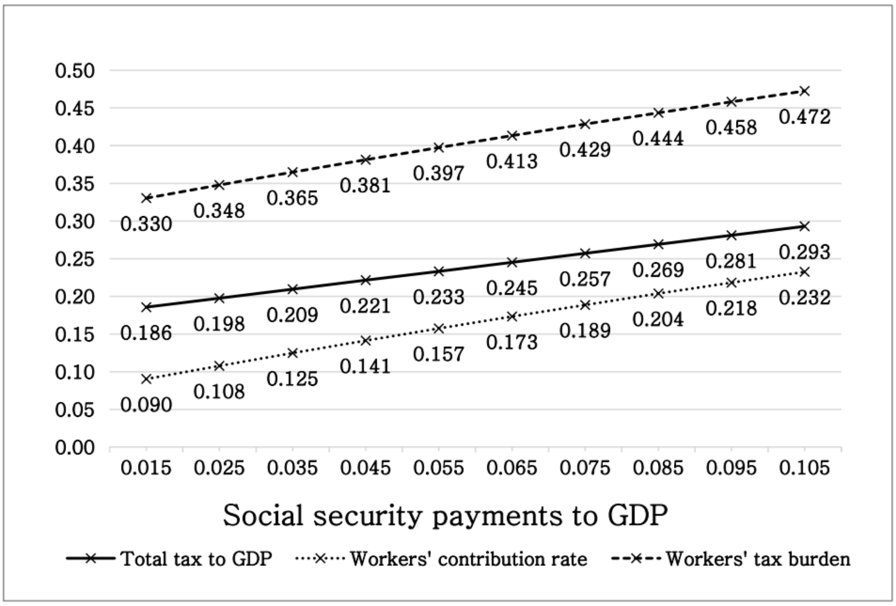

Figure 3-3. Effects of social security payments on total tax, workers' tax burden, and workers' contribution rate

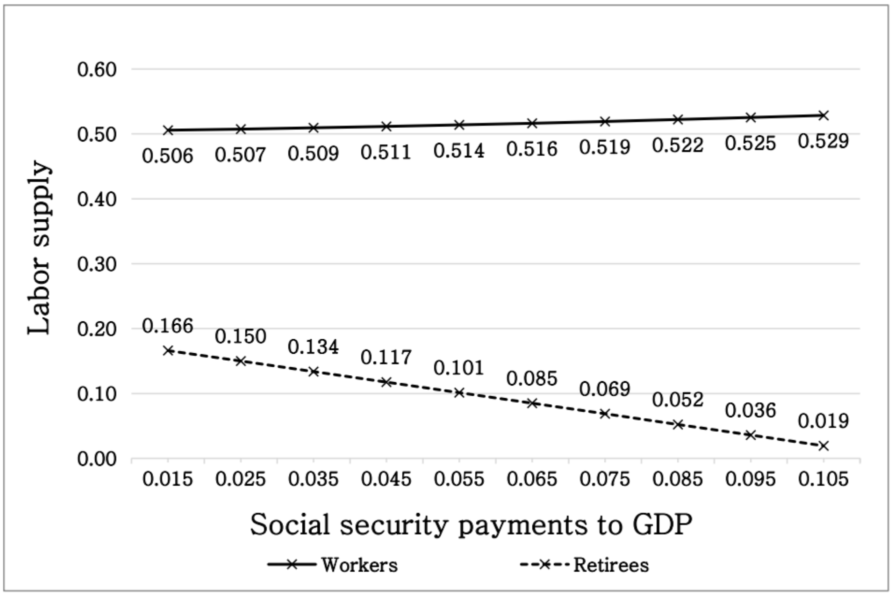

Figure 3-4. Effects of social security payments on labor supply

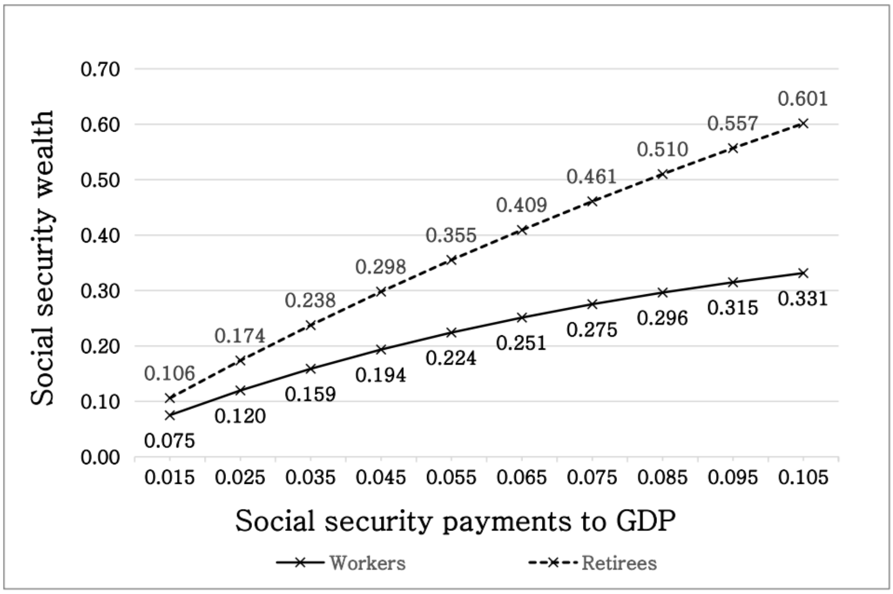

Figure 3-5. Effects of social security payments on social security wealth

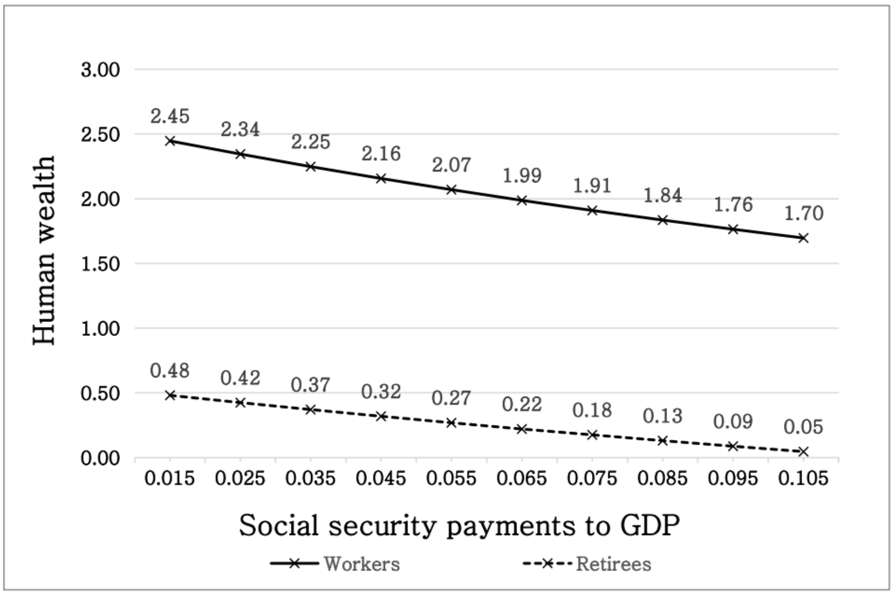

Figure 3-6. Effects of social security payments on human wealth

Table 3-3. Effects of social security payments on steady-state of total tax, workers' tax burden, and contribution rate

Social security payments (% of GDP)	Total tax (% of GDP)	Total tax (% of labor income)	Contribution rate (% of labor income)
1.45%	18.6%	33.0%	9.0%
2.45%	19.8%	34.8%	10.8%
3.45%	20.9%	36.5%	12.5%
4.45%	22.1%	38.1%	14.1%
5.45%	23.3%	39.7%	15.7%
6.45%	24.5%	41.3%	17.3%
7.45%	25.7%	42.9%	18.9%
8.45%	26.9%	44.4%	20.4%
9.45%	28.1%	45.8%	21.8%
10.45%	29.3%	47.2%	23.2%
△9.0%p	△10.7%p	△14.2%p	△14.2%p

Notes: The increase in the proportion of social security payments to GDP from 1.45% to 10.45% reflects the increase in the proportion of expenditure on national pension benefits to GDP from 1.0% to 10.0%, given the proportion of basic pension to GDP is constant at 0.44%

In Figure 3-7 and Figure 3-8, the proportion of total consumption to GDP increases from 58.7% to 64.2% by about 9.5%. The increase in the consumption–to–GDP ratio of retirees is much larger than that of workers due to the increased share of financial assets and social security wealth. The propensity to consume out of wealth of both groups increases, as shown in Figure 3–9. The consumption rate of retirees is always larger than that of workers, and the ratio of the two propensities to consume gradually decreases but is always larger than one.

For the whole economy, the capital per efficiency unit of labor falls from 3.25 to 2.22 by nearly 21.3 %, as shown in Figure 3–10. If the rise in the number of retirees because of longer retirees' life expectancy increases the national pension payments, capital may rise by increasing the incentive to save.

Consequently, the effects of increasing the ratio of social security payments to GDP on the economy are summed up as follows. The effects of an asset transfer and an increase in social security wealth lead to an increase in retirees' consumption, and this also increases the percentage of the total consumption to GDP. Retirees' labor supply decreases sharply due to increased financial assets and social security benefits, while that of workers slightly increases as consumption demand expands. However, workers' increased tax burden on labor income by increasing the social security payments–to–GDP ratio has a negative effect on human wealth. An increase in pension benefits that do not reflect retirees living longer does not result in an individual's saving incentive; thus, capital stock falls, and the gross return on capital rises.

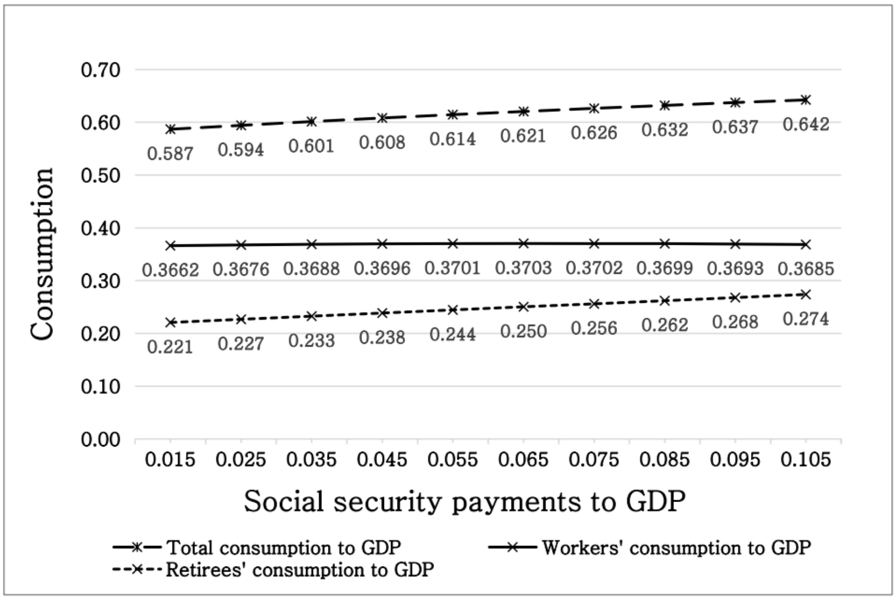

Figure 3−7. Effects of social security payments on consumption

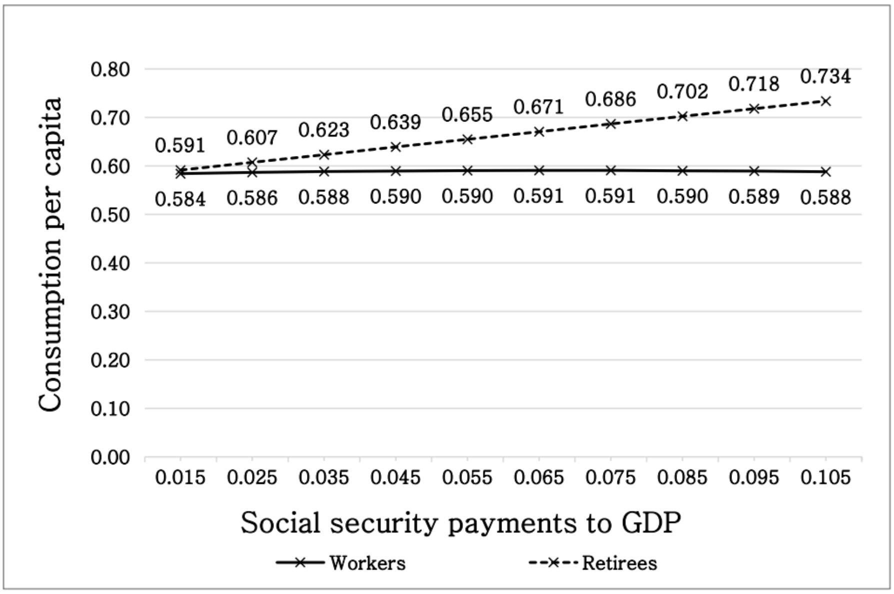

Figure 3−8. Effects of social security payments on consumption per capita

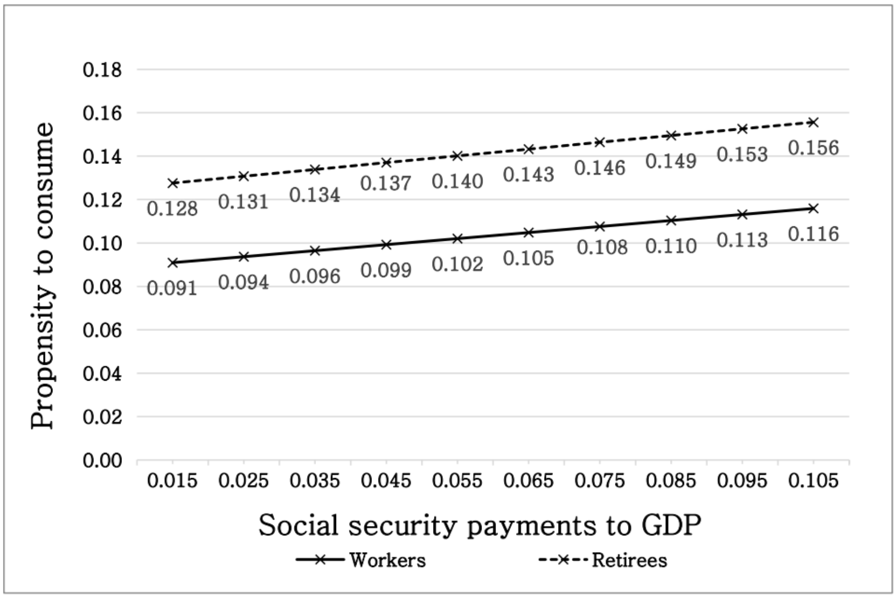

Figure 3–9. Effects of social security payments on propensity to consume

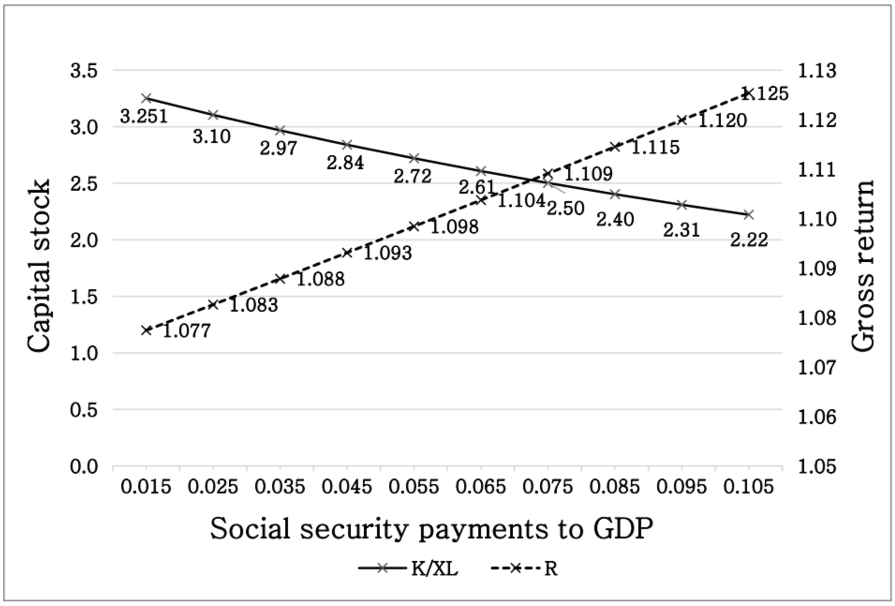

Figure 3–10. Effects of social security payments on capital and gross return

2) Effects of aging

This section analyzes the impact of the continuing rise in retirees' life expectancy on the economy as population aging intensifies. According to the latest population projections for Korea published in 2019, life expectancy in South Korea is projected to rise by 8.8 years to 88.5 for males and 6.0 years to 91.7 for females by 2067. Furthermore, total life expectancy is projected to rise by 7.4 years to 90.1. Table 3-4 shows changes in the survival rate of retirees and changes in population composition, reflecting this trend of increasing life expectancy.

Table 3-4. Effects of an aging on retiree's survival rate and population structure

Life expectancy at birth (unit:years)	Life expectancy of a retiree (unit:years)	Retiree's probability of survival(γ)	Percentage of each group	
			Workers	Retirees
83	12	0.917	62.7%	37.3%
84	13	0.923	61.0%	39.0%
85	14	0.929	59.4%	40.6%
86	15	0.933	58.0%	42.0%
87	16	0.938	56.6%	43.4%
88	17	0.941	55.3%	44.7%
89	18	0.944	54.1%	45.9%
△6	△6	△0.027	▽8.56%p	△8.56%p

If retirees' life expectancy increases without an adjustment of the retirement age, it relatively increases the proportion of the retirement population. With a 6-year increase in retiree's life expectancy, the probability of survival of retirees increases from 0.917 to 0.944, and the percentage of each group decreases by 8.56%p for workers and increases by 8.56%p for retirees. The proportion of workers to total population decreases from 62.7% to 54.1%, and that of retirees to total population increases from 37.3% to 45.9%.

The social security wealth of both workers and retirees increases, not because of an increase in social security benefits per capita, but because of an increased retirement period. The ratio of social security wealth to GDP of workers increases from 0.075 to 0.085 and that of retirees increases from 0.106 to 0.153. (see Figure 3-11)

As an individual lives longer than before, the labor supply of both workers and retirees increases more than before. In Figure 3-12, workers' labor supply increases from 50.6% to 53.2% by 5.1% and retirees' labor supply increases from 16.6% to 22.9% by 37.9% as the life expectancy of a retiree increases by 6 years. Thus, for both workers and retirees, the ratio of human wealth to GDP also increases, as shown in Figure 3-13. Moreover, asset transfers from workers to retirees increase as the proportion of retirees increases. The share of assets for retirees increases more than before, from 43.2 % to 52.3 %, while the share of assets for workers decreases from 56.8% to 47.7. However, the share of assets per retiree is maintained at 1.28~1.29 without significant change due to the increase in the number of retirees although there is an increase in the total share of assets.

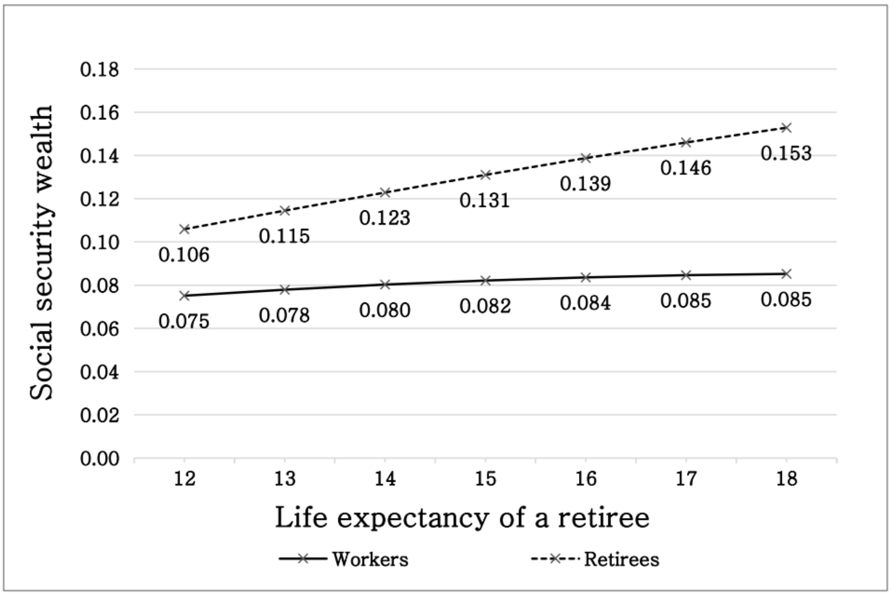

Figure 3-11. Effects of life expectancy on social security wealth

Figure 3-12. Effects of life expectancy on labor supply

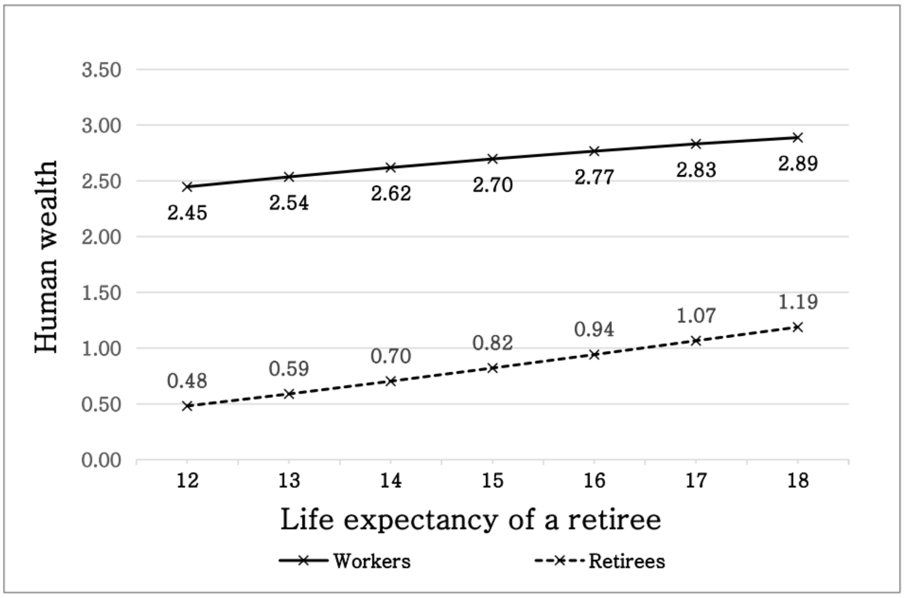

Figure 3−13. Effects of life expectancy on human wealth

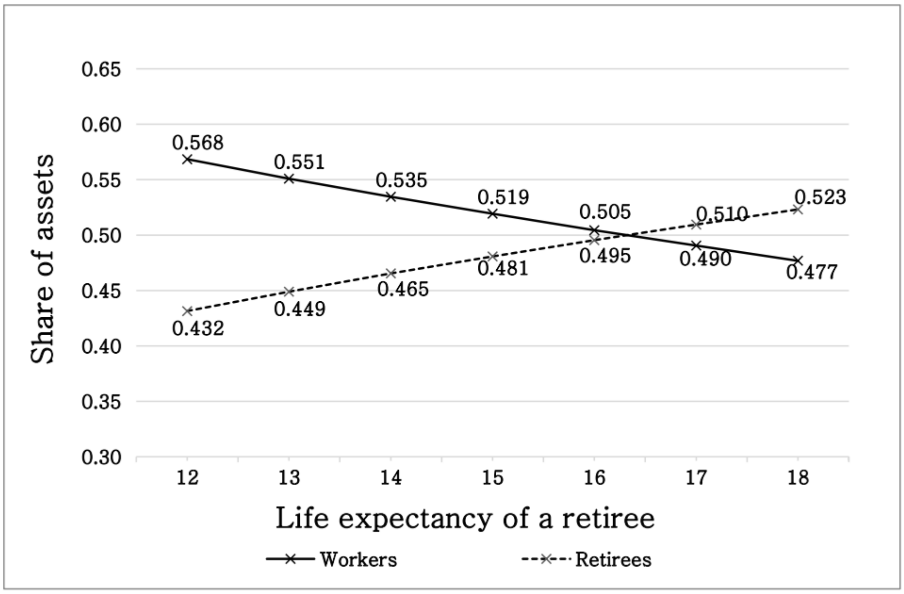

Figure 3−14. Effects of life expectancy on share of assets

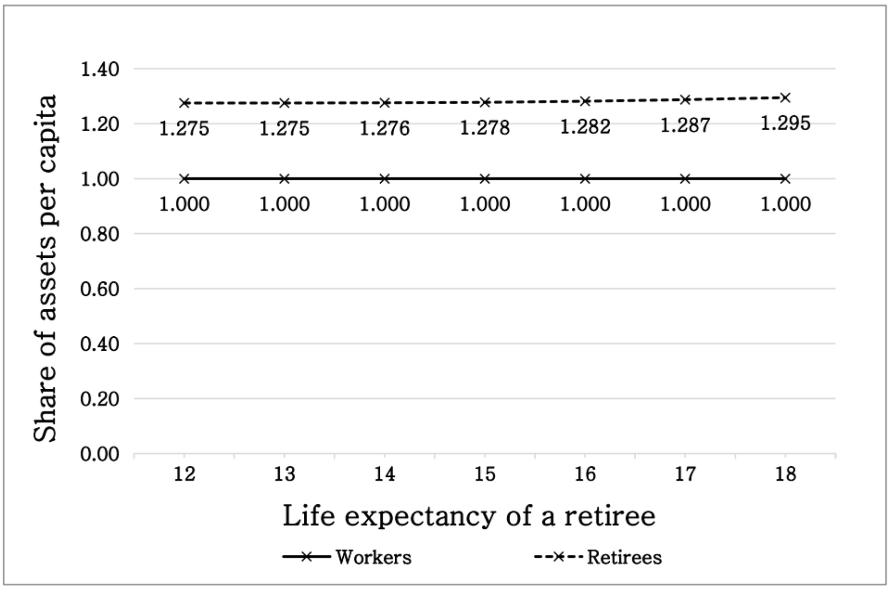

Figure 3-15. Effects of life expectancy on share of assets per capita

As life expectancy increases, the period in retirement becomes longer, reducing both a worker's and a retiree's consumption rate for their life after retirement. It leads to a decrease in total consumption to GDP from 58.7% to 55.6% despite the increase in the ratio of social security wealth and human wealth to GDP. In the same context, an individual's incentive to save for old age leads to a rise in capital stock to GDP from 3.25 to 3.89 and a reduction in gross return on capital from 1.077 to 1.058.

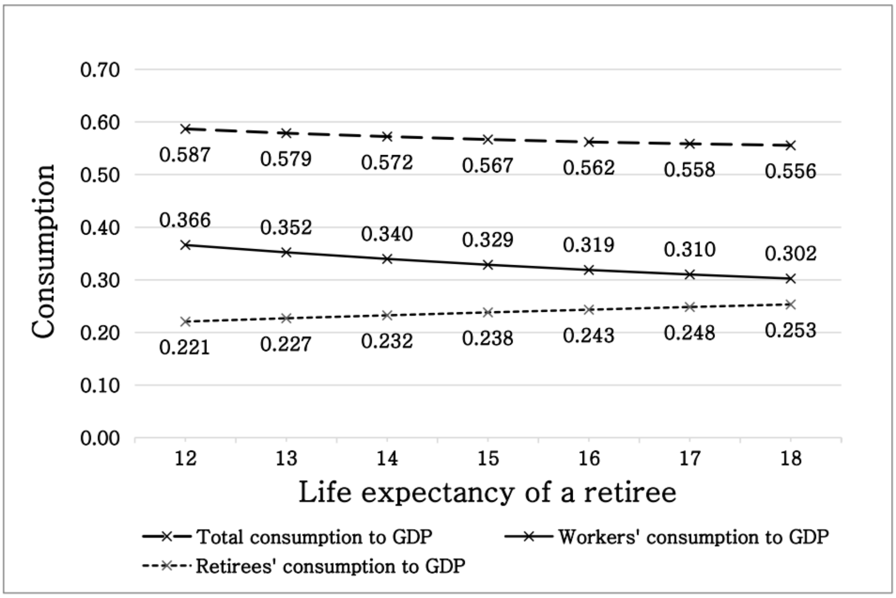

Figure 3-16. Effects of life expectancy on consumption

Figure 3-17. Effects of life expectancy on propensity to consumption

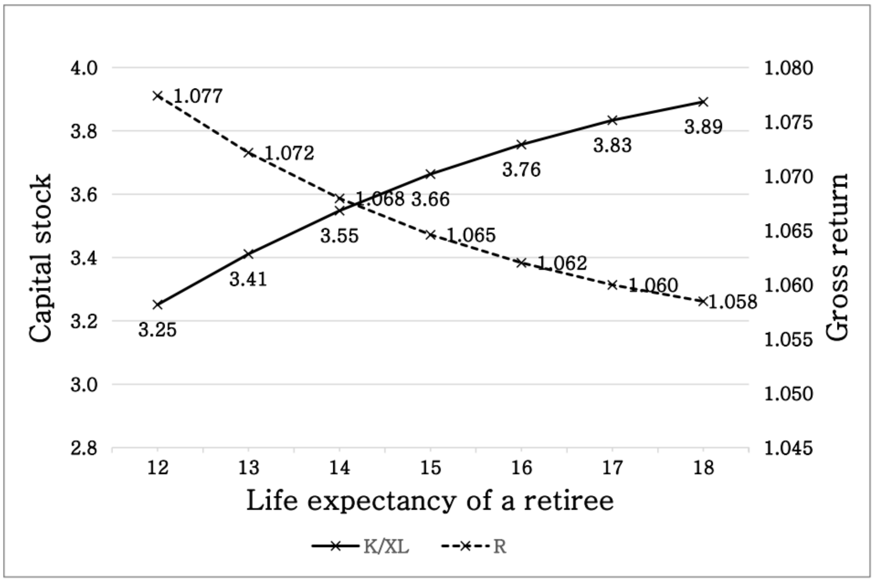

Figure 3-18. Effects of life expectancy on capital stock and gross return

3) Effects of the retirement age extension

According to OECD statistics in 2019 regarding pension systems, Korea's current retirement age is 3.2 years lower for men and 2.5 years lower for women than the OECD average, and expected years in retirement are 4.9 years lower for men and 6.2 years lower for women than the OECD average.

Although the concept of statutory retirement ages and retirement ages in a pension system differs slightly worldwide, these statistics are used to conduct analyses. Realistically, however, it is assumed that Korea's retirement age and the life expectancy of retirees gradually reach half the OECD average since extending the retirement age is a matter of policy decision making, making it difficult to realize in a short period. Therefore, the scenario assumes that the retirement age is extended by 1.5 years, with 0.25 years per year, while life expectancy increases by six years. The results show that the proportion of retirees increases by 5.70%p, less than the 8.56%p increase given the aging effect. In other words, extending the retirement age increases the proportion of workers and reduces that of retirees.

Since there is still an aging effect even if the statutory retirement age of workers is extended, the steady−state of the economy is similar to before the retirement age extension. As the aging population progresses, the differences between before and after the retirement age extension policy are as follows.

First of all, variation in the length of working time by the retirement age extension has an alleviation effect on the variation in labor supply of both workers and retirees. In Figure 3−20, compared to labor supply with only the aging effect in Figure 3−12, the increment in the labor supply of both workers and retirees is slightly smaller because it is possible to prepare for retirement with less labor due to more extended working periods by the retirement extensions.

Second, after adjusting the retirement age even with the same life expectancy, the fluctuation in the share of assets for workers and retirees is smaller than before. Considering only the aging effect, the retirees' share of assets reverses that of workers when the life expectancy of retirees is 17 years or more, as shown in Figure 3−14. However, after the retirement age is extended, the gap in both share of assets between workers and retirees is smaller than before as shown in Figure 3−22.

Third, as an effect of extending the retirement age, the fluctuation in the ratio of consumption to GDP is more significant than before. The ratio of total consumption to GDP decreases even more after extending the retirement age than when there is only an aging effect. Workers and retirees increase their leisure time and further reduce consumption with a relatively less increased labor supply than when there is only an aging effect as the retirement age is extended. As a result of the working period extension due to retirement age adjustment, capital stock throughout the economy increases from 3.25 to 4.07, which is higher than the steady−state of capital stock when there is only an aging effect.

Table 3−5. Simultaneous effects of an aging and a retirement age extension on transition probability and population structure

(unit: years, %)

Life expectancy at birth	Retirees			Workers		Percentage of each group	
	Life expecta−ncy	Survival rate	Retirement age	Working period expectancy	probability of remaining in labor force	Workers	Retirees
83	12	91.67%	60.00	18.00	94.44%	62.7%	37.3%
84	13	92.31%	60.25	18.25	94.52%	61.3%	38.7%
85	14	92.86%	60.50	18.50	94.59%	60.1%	39.9%
86	15	93.33%	60.75	18.75	94.67%	59.0%	41.0%
87	16	93.75%	61.00	19.00	94.74%	57.9%	42.1%
88	17	94.12%	61.25	19.25	94.81%	57.0%	43.0%
89	18	94.44%	61.50	19.50	94.87%	56.1%	43.9%
△6	△6	△2.78%	△1.5	△1.0	△0.43%	▽5.70%p	△5.70%

Figure 3-19. Simultaneous effects of an aging and a retirement extension on social security wealth

Figure 3-20. Simultaneous effects of an aging and a retirement extension on labor supply

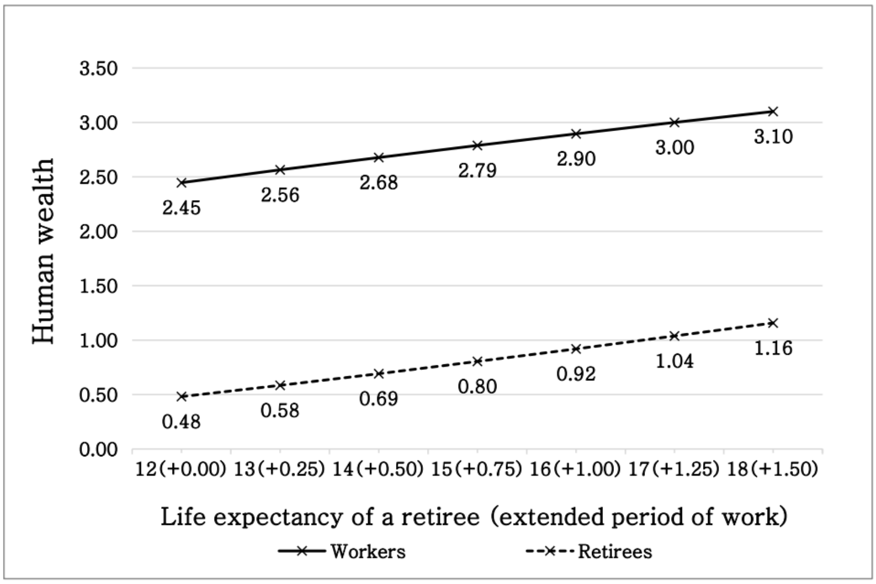

Figure 3-21. Simultaneous effects of an aging and a retirement extension on human wealth

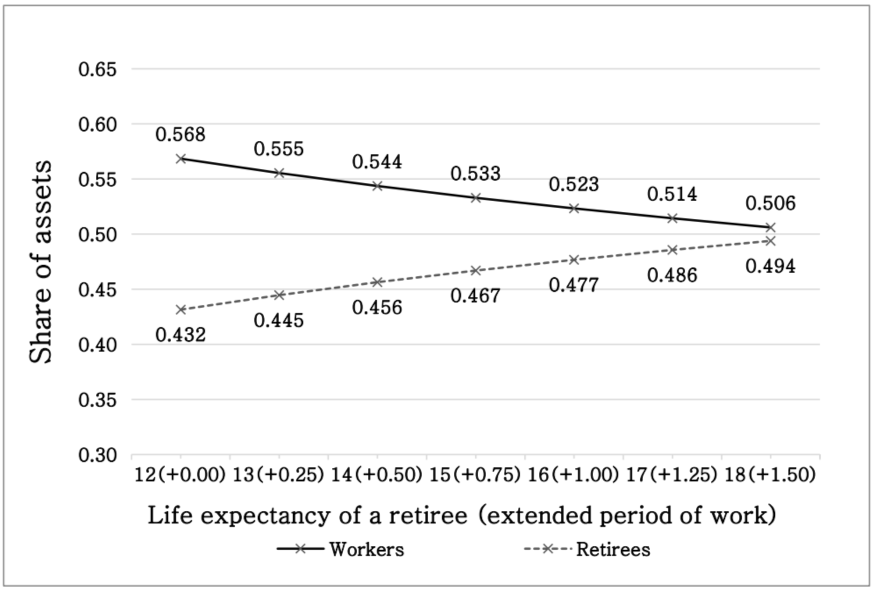

Figure 3-22. Simultaneous effects of an aging and a retirement extension on share of assets

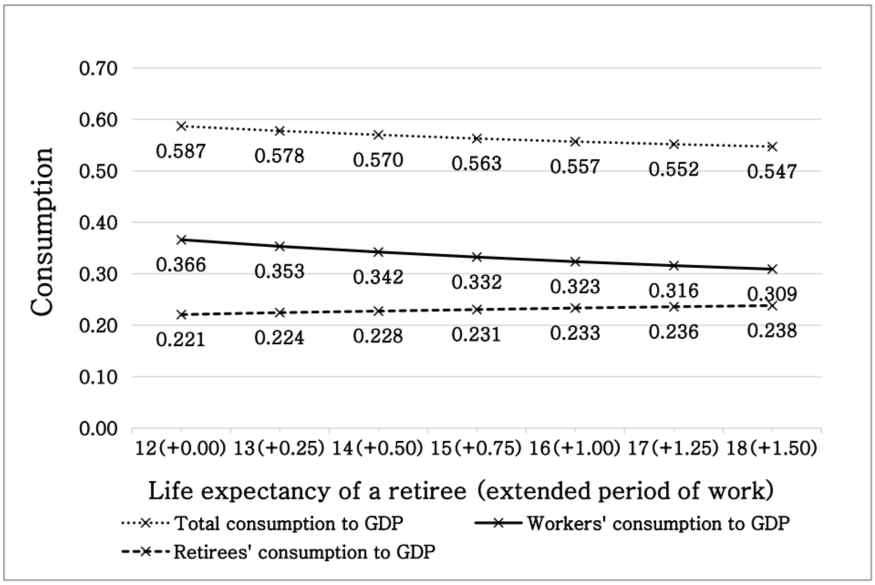

Figure 3-23. Simultaneous effects of an aging and a retirement extension on consumption

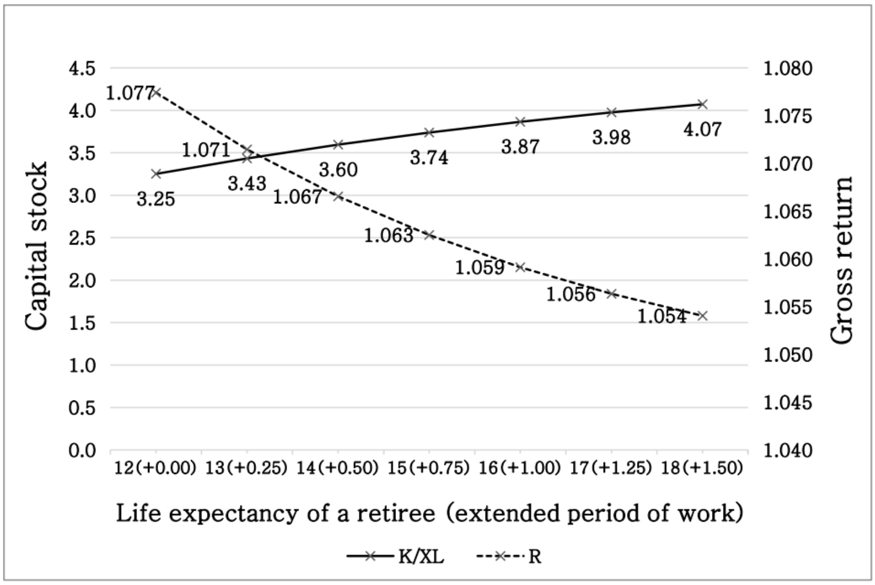

Figure 3-24. Simultaneous effects of an aging and a retirement extension on capital stock and gross return

3. Sub−conclusion

This chapter analyzes the impact of social policy, aging, and retirement age extension on the Korean economy under reasonable parameters. The conclusions that can be drawn from this study are as follows.

First, increasing the ratio of social security payments to GDP results in asset transfers from workers to retirees because the source of funds to pay retirees more benefits is taxes collected from workers. Retirees, who have become more prosperous than before, reduce labor supply and increase the consumption−to−GDP ratio. However, for workers, the increased tax burden has a negative effect on their financial wealth and human wealth. There is an increase in pension benefits that does not reflect retirees living longer and does not result in an individual's incentive to save; thus, the ratio of total consumption to GDP rises, and the capital stock falls.

Second, as life expectancy increases, the period in retirement becomes longer, which reduces both a worker's and a retiree's propensity to consume out of wealth in preparation for their life after retirement, which leads to a decrease in the proportion of total consumption to GDP despite the increase in social security wealth and human wealth to GDP. In the same context, an individual's incentive to save for old age leads to a rise in capital stock to GDP and a reduction in gross return on capital. After the retirement age extension policy is implemented, the length of working time has an alleviation effect on the labor supply variation of both workers and retirees. That is because in the elastic labor markets, as the working period is extended, it is possible to prepare for retirement with less labor.

In addition, the gap in the share of assets per capita between groups has narrowed compared to the previous one as the fluctuation in the share of financial assets has become smaller than before.

On the contrary, the decrease in consumption ratio to GDP is greater than before since workers and retirees increase leisure time and further reduce consumption with a relatively less increased labor supply than when there is only an aging effect. As a result of the working period's extension due to retirement age adjustment, capital stock throughout the economy increases higher than the steady−state of capital stock when there is only an aging effect.

In this chapter, Section 1 and Section 2−2 and 2−3 were published in Communications for Statistical Applications and Methods on November 30, 2020. See Lee and Son (2020). Section 2−1 was published in Korea Risk Management Association on December 30, 2020. See Son (2020) for more information.

Chapter 4. Modified Gertler's OLG Model

1. Introduction

Though Gertler's life cycle model has many advantages such as tractability, parsimony, and flexibility useful for economic analysis, it should be improved by modifying two factors in valuations of workers' decision problems. First, an adjustment factor Ω should be rederived by applying the Envelope Theorem. Second, the inconsistency of valuations in non−financial assets to the transition from employment to retirement should be improved.

Therefore, I rigorously derive these two factors, modify Gertler's model, and compare it with Gertler's model in Chapter 2. The previous Chapter 2 provided a clear and detailed derivation of agents' decisions, including elastic labor supply.

The critical differences between Gertler's model and the proposed model are as follows: first, the definition and role of adjustment factor differ in the two models. Second, the worker's non−financial assets valuations are consistent in our proposed model but not in Gertler's model. In particular, in Gertler's model, social security assets valuation contains an ambiguous transition factor ε / ψ. It overestimates the steady−state value of social security wealth compared with the result of the proposed model.

More details on the derivation of the modified model are described in Section

2. Section 3 provides an interpretation of the results of Gertler's model and the modified model and compares them. The key differences between the two models are summed up in Table 4−1 and Table 4−2, and a comparison of the steady−state values from the modified model with those from Gertler's model is given in Table 4−3.

2. Modification of model

This section presents the modified model by rigorously deriving the two factors in valuations in the worker's decision problems. The basic assumptions such as population structure, production, and the solutions to the retiree's decision problems are consistent with those described in Chapter 2.

This chapter focuses on the worker's decision problems. A worker maximizes utility function (4.2.1) subject to the budget constraint (4.2.2) as follows:

$$\text{Max} \quad V_t^w = \left\{ [(C_t^w)^{\upsilon}(1-l_t^w)^{1-\upsilon}]^{\rho} + \beta[\omega V_{t+1}^w + (1-\omega)V_{t+1}^r]^{\rho} \right\}^{1/\rho} \tag{4.2.1}$$

$$\text{subject to} \quad A_{t+1}^w = R_t A_t^w + W_t\, l_t^w - C_t^w . \tag{4.2.2}$$

Unlike a retiree, a worker does not receive social security benefits and pays taxes. The worker’s financial assets A^w evolve according to (4.2.2), where R_t is the gross return on assets, $W_t\, l_t^w$ is post−tax labor income, and C_t^w is consumption.

The optimization problem can be written as

$$L^w = V_t^w - \mu(C_t^w + A_{t+1}^w - R_t A_t^w - W_t l_t^w) . \tag{4.2.3}$$

Let us partially differentiate L^w with respect to C_t^w, l_t^w, and A_{t+1}^w. From the four partial derivatives, the following equations yield:

$$1 - l_t^w = \tfrac{1-\upsilon}{\upsilon} C_t^w / W_t , \tag{4.2.4}$$

$$\upsilon (C_t^w)^{\upsilon\rho-1}(1-l_t^w)^{(1-\upsilon)\rho} = [\omega V_{t+1}^w + (1-\omega)V_{t+1}^r]^{\rho-1} \beta [\omega \tfrac{\partial V_{t+1}^w}{\partial A_{t+1}^w} + (1-\omega)\tfrac{\partial V_{t+1}^r}{\partial A_{t+1}^r}] , \tag{4.2.5}$$

$$\mu = \upsilon (C_t^w)^{\upsilon\rho-1}(1-l_t^w)^{(1-\upsilon)\rho}(V_t^w)^{1-\rho} . \tag{4.2.6}$$

Applying the Envelope Theorem with parameters A_t^w and A_t^r,

$$\tfrac{dV_t^w}{dA_t^w} = \tfrac{\partial L^w}{\partial A_t^w} = \mu R_t = R_t \upsilon (C_t^w)^{\upsilon\rho-1}(1-l_t^w)^{(1-\upsilon)\rho}(V_t^w)^{1-\rho} , \tag{4.2.7}$$

$$\tfrac{dV_t^r}{dA_t^r} = \tfrac{\partial L^r}{\partial A_t^r} = \mu \tfrac{R_t}{\gamma} = \tfrac{R_t}{\gamma} \upsilon (C_t^r)^{\upsilon\rho-1}(1-l_t^r)^{(1-\upsilon)\rho}(V_t^r)^{1-\rho} . \tag{4.2.8}$$

From the above two equations, the following equations are obtained.

$$\tfrac{\partial V_{t+1}^w}{\partial A_{t+1}^w} = R_{t+1} \upsilon (C_{t+1}^w)^{\upsilon\rho-1}(1-l_{t+1}^w)^{(1-\upsilon)\rho}(V_{t+1}^w)^{1-\rho} , \tag{4.2.9}$$

$$\tfrac{\partial V_{t+1}^r}{\partial A_{t+1}^r} = \tfrac{R_{t+1}}{\gamma} \upsilon (C_{t+1}^r)^{\upsilon\rho-1}(1-l_{t+1}^r)^{(1-\upsilon)\rho}(V_{t+1}^r)^{1-\rho} . \tag{4.2.10}$$

On the other hand, in Gertler (1999), $\tfrac{\partial V_{t+1}^r}{\partial A_{t+1}^r}$ was derived by (Appendix (A.1.11))

$$\tfrac{\partial V_{t+1}^r}{\partial A_{t+1}^r} = R_{t+1} \upsilon (C_{t+1}^r)^{\upsilon\rho-1}(1-l_{t+1}^r)^{(1-\upsilon)\rho}(V_{t+1}^r)^{1-\rho} . \tag{4.2.11}$$

The difference between (4.2.10) and (4.2.11) results in different adjustment factors for each model, such as equations (4.2.17) and (2.3.15).

Now, let us guess the form of V_t^w and V_t^r as follows:

$$V_t^w = (\pi_t)^{-\frac{1}{\rho}} (C_t^w)^{\upsilon} (1-l_t^w)^{1-\upsilon} , \tag{4.2.12}$$

$$V_t^r = (\varepsilon_t \pi_t)^{-\frac{1}{\rho}} (C_t^r)^{\upsilon} (1-l_t^r)^{1-\upsilon}, \tag{4.2.13}$$

where π_t and $\varepsilon_t \pi_t$ denote the propensity to consume out of wealth for a worker and a retiree, respectively, and ε_t means the retiree's consumption elasticity.

Equations (4.2.12) and (4.2.13) may be rewritten using equations (4.2.4) and (2.3.5), which are the labor supply curve of a worker and a retiree in Gertler (1999), as follows:

$$V_{t+1}^w = (\pi_{t+1})^{-\frac{1}{\rho}} C_{t+1}^w (\tfrac{1-\upsilon}{\upsilon W_{t+1}})^{1-\upsilon}, \tag{4.2.14}$$

$$V_{t+1}^r = (\varepsilon_{t+1} \pi_{t+1})^{-\frac{1}{\rho}} C_{t+1}^r (\tfrac{1-\upsilon}{\upsilon W_{t+1}^r})^{1-\upsilon}. \tag{4.2.15}$$

If (4.2.9), (4.2.10), (4.2.14), and (4.2.15) are substituted into (4.2.5), following equation is obtained as

$$(C_t^w)^{\rho-1} = (\tfrac{W_t}{W_{t+1}})^{(1-\upsilon)\rho} [\omega C_{t+1}^w + (1-\omega)(\varepsilon_{t+1})^{-\frac{1}{\rho}} C_{t+1}^r (\tfrac{W_{t+1}}{W_{t+1}^r})^{1-\upsilon}]^{\rho-1} \beta R_{t+1} [\omega + (1-\omega)\tfrac{1}{\gamma} (\tfrac{W_{t+1}}{W_{t+1}^r})^{1-\upsilon} (\varepsilon_{t+1})^{-\frac{1-\rho}{\rho}}]. \tag{4.2.16}$$

Let us define that $\xi = W_{t+1}^r / W_{t+1}$, $\chi = (1/\xi)^{1-\upsilon}$, and $\sigma = 1/(1-\rho)$. Then Ω can be defined as

$$\Omega_{t+1} = \omega + (1-\omega)\tfrac{1}{\gamma} (\varepsilon_{t+1})^{\frac{1}{1-\sigma}} \chi. \tag{4.2.17}$$

On the other hand, in Gertler (1999), Ω^G [1] was given by

$$\Omega_{t+1}^G = \omega + (1-\omega)(\varepsilon_{t+1})^{\frac{1}{1-\sigma}} \chi. \tag{2.3.15}$$

---

[1] To distinguish the adjustment factor in the proposed model from that in Gertler's model, we mark G in the superscript of Ω_{t+1} in Gertler's model.

The definitions of the risk adjustment factor in the two models are different, and Ω plays a role in adjusting transition probabilities in valuations, as shown in (4.2.36) and (4.2.37), while Ω^G plays a role in adjusting the discounting factor, as shown in (2.3.11) and (2.3.12).

By substituting (4.2.17) for (4.2.16), the following consumption Euler equation is obtained.

$$\omega C_{t+1}^{w} + (1-\omega)\chi(\varepsilon_{t+1})^{\frac{\sigma}{1-\sigma}} C_{t+1}^{r} = [(\tfrac{W_t}{W_{t+1}})^{1-\upsilon}]^{\sigma-1}(R_{t+1}\Omega_{t+1}\beta)^{\sigma} C_t^{w} \quad . \tag{4.2.18}$$

Now, let us guess the form of consumption functions as follows:

$$C_t^{w} = \pi_t(R_t A_t^{w} + H_t^{w} + S_t^{w}), \tag{4.2.19}$$

$$C_t^{r} = \varepsilon_t \pi_t(\tfrac{R_t}{\gamma} A_t^{r} + H_t^{r} + S_t^{r}). \tag{4.2.20}$$

An individual's consumption depends upon the propensity to consume and the wealth at the end of time t. Wealth includes financial wealth, and human wealth H_t^i, the present value of the future labor income, and social security wealth S_t^i, the present value of the future social security benefits.

From (4.2.19), (4.2.20), the following equations are obtained as

$$C_{t+1}^{w} = \pi_{t+1}(R_{t+1}A_{t+1}^{w} + H_{t+1}^{w} + S_{t+1}^{w}), \tag{4.2.21}$$

$$C_{t+1}^{r} = \varepsilon_{t+1}\pi_{t+1}(\tfrac{R_{t+1}}{\gamma} A_{t+1}^{r} + H_{t+1}^{r} + S_{t+1}^{r}). \tag{4.2.22}$$

Substituting (4.2.19), (4.2.21), and (4.2.22) into (4.2.18), then

$$\begin{aligned}&\pi_{t+1}[\omega(R_{t+1}A_{t+1}^{w} + H_{t+1}^{w} + S_{t+1}^{w}) + (1-\omega)(\varepsilon_{t+1})^{\frac{1}{1-\sigma}}\chi(\tfrac{R_{t+1}}{\gamma}A_{t+1}^{r} + H_{t+1}^{r} + S_{t+1}^{r})]\\ &= [(\tfrac{W_t}{W_{t+1}})^{1-\upsilon}]^{\sigma-1}(R_{t+1}\Omega_{t+1}\beta)^{\sigma}\pi_t(R_t A_t^{w} + H_t^{w} + S_t^{w}).\end{aligned} \tag{4.2.23}$$

Next, by placing (4.2.12), (4.2.14), and (4.2.15) into the worker's value function (4.2.1), then

$$[(\pi_t)^{-\frac{1}{\rho}} C_t^w (\tfrac{1-\upsilon}{\upsilon W_t})^{1-\upsilon}]^\rho = [C_t^w (\tfrac{1-\upsilon}{\upsilon W_t})^{1-\upsilon}]^\rho + \beta[\omega(\pi_{t+1})^{-\frac{1}{\rho}} C_{t+1}^w (\tfrac{1-\upsilon}{\upsilon W_{t+1}})^{1-\upsilon} + (1-\omega)(\varepsilon_{t+1}\pi_{t+1})^{-\frac{1}{\rho}} C_{t+1}^r (\tfrac{1-\upsilon}{\upsilon W_{t+1}^r})^{1-\upsilon}]^\rho . \tag{4.2.24}$$

Equation (4.2.24) can be written as

$$(\pi_t)^{-1} = 1 + \beta \left\{ (\tfrac{W_t}{W_{t+1}})^{1-\upsilon} [\omega C_{t+1}^w + (1-\omega)\chi(\varepsilon_{t+1})^{\frac{\sigma}{1-\sigma}} C_{t+1}^r] / C_t^w \right\}^{\frac{\sigma-1}{\sigma}} (\pi_{t+1})^{-1}, \tag{4.2.25}$$

and following equation using (4.2.17) is obtained.

$$\pi_t = 1 - [(\tfrac{W_t}{W_{t+1}})^{1-\upsilon} R_{t+1}\Omega_{t+1}]^{\sigma-1} \beta^\sigma (\pi_t / \pi_{t+1}) . \tag{4.2.26}$$

From (4.2.26),

$$\pi_{t+1} = [(\tfrac{W_t}{W_{t+1}})^{1-\upsilon} R_{t+1}\Omega_{t+1}]^{\sigma-1} \beta^\sigma [\pi_t / (1-\pi_t)], \tag{4.2.27}$$

and multiply both sides by the same equations as $R_{t+1}\Omega_{t+1}(1-\pi_t)(R_t A_t^w + H_t^w + S_t^w)$,

then

$$\pi_{t+1}[R_{t+1}\Omega_{t+1}(1-\pi_t)(R_t A_t^w + H_t^w + S_t^w)] = [(\tfrac{W_t}{W_{t+1}})^{1-\upsilon}]^{\sigma-1} (R_{t+1}\Omega_{t+1}\beta)^\sigma \pi_t (R_t A_t^w + H_t^w + S_t^w) . \tag{4.2.28}$$

Here, note that the RHS of (4.2.28) is equal to the RHS of (4.2.23). Hence,

$$\begin{aligned} & R_{t+1}\Omega_{t+1}(1-\pi_t)(R_t A_t^w + H_t^w + S_t^w) \\ & = \omega(R_{t+1}A_{t+1}^w + H_{t+1}^w + S_{t+1}^w) + (1-\omega)(\varepsilon_{t+1})^{\frac{1}{1-\sigma}} \chi (\tfrac{R_{t+1}}{\gamma} A_{t+1}^r + H_{t+1}^r + S_{t+1}^r). \end{aligned} \tag{4.2.29}$$

If both sides of the equation (4.2.29) are divided by $R_{t+1}\Omega_{t+1}$, then

$$(1-\pi_t)(R_t A_t^w + H_t^w + S_t^w) = \tfrac{\omega}{R_{t+1}\Omega_{t+1}} (R_{t+1}A_{t+1}^w + H_{t+1}^w + S_{t+1}^w) + \frac{(1-\omega)\frac{1}{\gamma}(\varepsilon_{t+1})^{\frac{1}{1-\sigma}}\chi}{R_{t+1}\Omega_{t+1}} [R_{t+1}A_{t+1}^r + \gamma(H_{t+1}^r + S_{t+1}^r)]. \tag{4.2.30}$$

Using (4.2.17), (4.2.30) can be rewritten as

$$(1-\pi_t)(R_t A_t^w + H_t^w + S_t^w) = \tfrac{1}{R_{t+1}} \tfrac{\omega}{\Omega_{t+1}} (R_{t+1}A_{t+1}^w + H_{t+1}^w + S_{t+1}^w) + \tfrac{1}{R_{t+1}} (1 - \tfrac{\omega}{\Omega_{t+1}}) [R_{t+1}A_{t+1}^r + \gamma(H_{t+1}^r + S_{t+1}^r)]. \tag{4.2.31}$$

It is also possible to express the LHS of (4.2.31) as

$$(1-\pi_t)(R_tA_t^w + H_t^w + S_t^w) = (R_tA_t^w + W_tl_t^w - C_t^w) + (H_t^w - W_tl_t^w) + (S_t^w - 0)\,. \qquad (4.2.32)$$

Then, equation (4.2.33) holds.

$$\begin{aligned}&(R_tA_t^w + W_tl_t^w - C_t^w) + (H_t^w - W_tl_t^w) + (S_t^w - 0)\\&= \tfrac{1}{R_{t+1}}\tfrac{\omega}{\Omega_{t+1}}(R_{t+1}A_{t+1}^w + H_{t+1}^w + S_{t+1}^w) + \tfrac{1}{R_{t+1}}(1-\tfrac{\omega}{\Omega_{t+1}})[R_{t+1}A_{t+1}^r + \gamma(H_{t+1}^r + S_{t+1}^r)]\end{aligned} \qquad (4.2.33)$$

which implies that the worker's wealth at the end of time t after cash flows have occurred becomes either the worker's wealth or the retiree's wealth at the end of time t +1.

It is evident from (4.2.2) that $A_{t+1}^w = A_{t+1}^r$,

$$R_tA_t^w + W_tl_t^w - C_t^w = \tfrac{\omega}{\Omega_{t+1}}A_{t+1}^w + (1-\tfrac{\omega}{\Omega_{t+1}})A_{t+1}^r\,. \qquad (4.2.34)$$

Thus, the equation for the worker's non−financial assets valuations is derived by

$$H_t^w + S_t^w = W_tl_t^w + \tfrac{1}{R_{t+1}}\tfrac{\omega}{\Omega_{t+1}}(H_{t+1}^w + S_{t+1}^w) + \tfrac{1}{R_{t+1}}(1-\tfrac{\omega}{\Omega_{t+1}})\gamma(H_{t+1}^r + S_{t+1}^r) \qquad (4.2.35)$$

and the following equations for valuations by asset also hold.

$$H_t^w = W_tl_t^w + \tfrac{1}{R_{t+1}}\tfrac{\omega}{\Omega_{t+1}}H_{t+1}^w + \tfrac{1}{R_{t+1}}(1-\tfrac{\omega}{\Omega_{t+1}})\gamma H_{t+1}^r\,, \qquad (4.2.36)$$

$$S_t^w = \tfrac{1}{R_{t+1}}\tfrac{\omega}{\Omega_{t+1}}S_{t+1}^w + \tfrac{1}{R_{t+1}}(1-\tfrac{\omega}{\Omega_{t+1}})\gamma S_{t+1}^r\,. \qquad (4.2.37)$$

As shown in (4.2.36) and (4.2.37), valuations of human wealth and social security wealth are consistent. When the worker remains in the labor force in the next period, social security and human wealth are evaluated by a factor $\tfrac{1}{R_{t+1}}\tfrac{\omega}{\Omega_{t+1}}$: the production of discounting factor $\tfrac{1}{R_{t+1}}$ and risk−adjusted probability $\tfrac{\omega}{\Omega_{t+1}}$. When the worker goes from work to retirement in the next period, social security wealth, as well as human wealth, is evaluated by a factor $\frac{1}{R_{t+1}}(1-\frac{\omega}{\Omega_{t+1}})$: the production of discounting factor $\frac{1}{R_{t+1}}$ and risk-adjusted probability $(1-\frac{\omega}{\Omega_{t+1}})$.

In the transition from a worker to a retiree, a retiree's survival rate is needed because a retiree can earn labor income and receive social security benefits conditional on surviving.

On the other hand, Gertler's valuations of human wealth and social security wealth at t are respectively given by:

$$H_t^w = W_t l_t^w + \frac{1}{R_{t+1}\Omega_{t+1}^G}\omega H_{t+1}^w + \frac{1}{R_{t+1}\Omega_{t+1}^G}(1-\omega)H_{t+1}^r, \tag{2.3.11}$$

$$S_t^w = \frac{1}{R_{t+1}\Omega_{t+1}^G}\omega S_{t+1}^w + \frac{1}{R_{t+1}\Omega_{t+1}^G}(1-\omega)\frac{\varepsilon_{t+1}}{\psi N_t}S_{t+1}^r, \tag{2.3.12}$$

where ψ is the ratio of retirees to workers, and N_t is the number of workers at time t. In Gertler (1999), as shown in (2.3.11), (2.3.12), valuations of non-financial assets are inconsistent because the valuation of social security wealth contains an ambiguous transition factor $\varepsilon_{t+1}/\psi N_t$.

Due to the changes in risk adjustment factor and worker's non-financial assets valuations in the modified model, aggregation functions also change as follows:

$$H_t^{w\bullet} = W_t L_t^w - T_t + \frac{1}{(1+n)R_{t+1}}\frac{\omega}{\Omega_{t+1}}H_{t+1}^{w\bullet} + \frac{1}{(1+n)R_{t+1}}(1-\frac{\omega}{\Omega_{t+1}})\gamma H_{t+1}^{r\bullet}, \tag{4.2.38}$$

$$S_t^{w\bullet} = \frac{1}{(1+n)R_{t+1}}\frac{\omega}{\Omega_{t+1}}S_{t+1}^{w\bullet} + \frac{1}{(1+n)R_{t+1}}(1-\frac{\omega}{\Omega_{t+1}})\gamma S_{t+1}^{r\bullet}. \tag{4.2.39}$$

Note that the aggregation function of workers' non-financial assets in Gertler's original model as follows:

$$H_t^{w\bullet} = W_t\ L_t^w - T_t + \tfrac{1}{(1+n)R_{t+1}\Omega_{t+1}^G}\omega H_{t+1}^{w\bullet} + \tfrac{1}{(1+n)R_{t+1}\Omega_{t+1}^G}(1-\omega)H_{t+1}^{r\bullet}, \tag{2.3.23}$$

$$S_t^{w\bullet} = \tfrac{1}{(1+n)R_{t+1}\Omega_{t+1}^G}\omega S_{t+1}^{w\bullet} + \tfrac{1}{R_{t+1}\Omega_{t+1}^G}\tfrac{\varepsilon_{t+1}}{(1+n)\psi}(1-\omega)S_{t+1}^{r\bullet}. \tag{2.3.24}$$

Note that also even in the modified model, retirees' equations for non−financial assets valuations are the same as those in Gertler's (1999) model and are as follows:

$$H_t^{r\bullet} = W_t^r L_t^r + \tfrac{1}{(1+n)R_{t+1}}\gamma H_{t+1}^{r\bullet}, \tag{4.2.40}$$

$$S_t^{r\bullet} = E_t^{\bullet} + \tfrac{1}{(1+n)R_{t+1}}\gamma S_{t+1}^{r\bullet}. \tag{4.2.41}$$

3. Comparison of results

I have so far modified two factors of workers' decision problems by rigorous derivation in Section 2. The main differences between Gertler (1999) and proposed are sum up in Table 4−1 and Table 4−2. In this section, steady state variables and equations are used. The method of deriving the steady state equations is specifically described in Section 3 of Chapter 2, and all steady state variables are denoted by the value of a variable relative to output. That is, h^w denotes the human wealth−to−output ratio for workers and s^w denotes the social security wealth−to−output ratio for workers in steady state.

As explained in the text and as shown in Table 4−1, in the proposed model, the definition of risk adjustment factor is modified by containing the annuity factor $1/\gamma$. Besides, Ω in the proposed model plays a role in adjusting transition probabilities while Ω^G in Gertler's model plays a role in adjusting the discounting factor.

As shown in Table 4−2, workers' valuations in non−financial assets are consistent in the proposed model by rigorous derivation, while those are inconsistent in Gertler’s model.

Table 4−1. Comparison of the adjustment factor and valuations between Gertler (1999) and the proposed model

<table>
<tr><td colspan="3"></td><td>Gertler (1999)</td><td>Proposed</td></tr>
<tr><td colspan="3">Adjustment factor</td><td>$\Omega^G = \omega + (1-\omega)\varepsilon^{\frac{1}{1-\sigma}}\chi$</td><td>$\Omega = \omega + (1-\omega)\frac{1}{\gamma}\varepsilon^{\frac{1}{1-\sigma}}\chi$</td></tr>
<tr><td rowspan="4">Valuations in non−financial assets</td><td rowspan="2">Maintaining the status of a worker in the next period</td><td>h^w</td><td rowspan="2">$\frac{1}{R\Omega^G}\omega$</td><td rowspan="2">$\frac{1}{R}\frac{\omega}{\Omega}$</td></tr>
<tr><td>s^w</td></tr>
<tr><td rowspan="2">Transition from a worker to a retiree in the next period</td><td>h^w</td><td>$\frac{1}{R\Omega^G}(1-\omega)$</td><td rowspan="2">$\frac{1}{R}(1-\frac{\omega}{\Omega})$</td></tr>
<tr><td>s^w</td><td>$\frac{1}{R\Omega^G}(1-\omega)\frac{\varepsilon}{\psi}$</td></tr>
</table>

Table 4−2. Comparison of the valuations of non−financial assets between Gertler (1999) and the proposed model

Gertler (1999)	$h^w = \alpha\frac{L^w}{L} - \tau + \frac{1}{R\Omega^G}\omega(1+x)h^w + \frac{1}{R\Omega^G}(1-\omega)(1+x)h^r$ $s^w = \frac{1}{R\Omega^G}\omega(1+x)s^w + \frac{1}{R\Omega^G}(1-\omega)\frac{\varepsilon}{\psi}(1+x)s^r$
Proposed	$h^w = \alpha\frac{L^w}{L} - \tau + \frac{1}{R}\frac{\omega}{\Omega}(1+x)h^w + \frac{1}{R}(1-\frac{\omega}{\Omega})\gamma(1+x)h^r$ $s^w = \frac{1}{R}\frac{\omega}{\Omega}(1+x)s^w + \frac{1}{R}(1-\frac{\omega}{\Omega})\gamma(1+x)s^r$

Table 4-3 shows the parameter values presented in Gertler (1999) and I calculate the steady state values of the basic exogenous parameters in both models.

Table 4-3. Description and value of parameters

Parameter	Description	Value
n	Workforce growth rate	0.01
ω	Probability of remaining in the labor force	0.977
γ	Probability of surviving of a retiree	0.9
υ	Preference parameter for consumption	0.4
$1-\upsilon$	Preference parameter for leisure	0.6
β	Subjective discount rate	1.0
ρ	Curvature parameter	-3
σ	Intertemporal elasticity of substitution	0.25
ξ	Productivity of a unit of labor supplied by a retiree relative to a worker	0.6
α	Labor income share ratio	0.667
δ	Capital depreciation rate	0.1
x	Growth rate of technology	0.01
b	Government debt to output	0.25
g	Government consumption to output expenditure	0.20
e	Social security payments to output	0.02

As a result of simulation of the model with modification factors, the steady state value of some variables changes, as shown in Table 4-4. First, the steady state value of the risk adjustment factor increases from 1.052 to 1.057 as the annuity factor γ is included. Thus, the adjustment factor value of Gertler (1999) is slightly underestimated than that obtained from the modified model.

Second, the steady state value of non-financial assets for workers in Gertler (1999) is overestimated than that in the modified model. In particular, workers' social security value-to-output ratio, has reduced significantly from 0.289 to

0.095. The significant difference in s^w is that there is an ambiguous transition factor ε/ψ in the valuation of social security wealth moving from work to retirement in Gertler's model. Since the value of ε is about 1.933 and the value of ψ is 0.209, ε/ψ overestimates the value of s^w. In addition, the capital per efficiency unit of labor rises from 3.458 to 3.491 and the gross return on capital falls from 1.046 to 1.045.

Table 4-4. Comparison of steady state values between Gertler (1999) and the proposed model

Variables	Gertler (1999)	Proposed	Variables	Gertler (1999)	Proposed
K/XL (Capital per efficiency unit of labor)	3.458	3.491	h^w (Human wealth-to-output ratio for workers)	4.134	4.079
R (Gross return)	1.046	1.045	h^r (Human wealth-to-output ratio for retirees)	0.121	0.121
Ω (Adjustment factor)	1.052	1.057	s^w (Social security wealth-to-output ratio for workers)	0.289	0.095
π (Propensity to consume out of wealth for workers)	0.065	0.067	s^r (Social security wealth-to-output ratio for retirees)	0.153	0.154
$\varepsilon\pi$ (Propensity to consume out of wealth for retirees)	0.126	0.125	$h^w + s^w$ (Non-financial assets-to-output ratio for workers)	4.424	4.174
ε (Retirees' consumption elasticity)	1.933	1.862			

4. Sub−conclusion

In this work, Gertler's OLG model is modified by rigorous derivations of adjustment factor and non−financial assets valuations. As a result, the improvements are as follows: first, a more accurate formula for Ω is derived. Second, the consistency in valuations of non−financial assets is ensured. In addition, it is found that the steady state value in Gertler's model is slightly underestimated for the risk adjustment factor and overestimated for the non−financial assets−to−output ratio more than in the proposed model. In particular, the difference in the value of s^w in the two models is significant because of the ambiguous transition factor ε/ψ included in the valuation of social security wealth when the worker goes from work to retirement in Gertler's model.

Chapter 5. Insurance Model

1. Introduction

This chapter attempts to incorporate non-life insurance into the modified Gertler's OLG model described in Chapter 4. It also sets out a framework in which the private and public sectors share the role of providing compensation for losses, assuming that the proportion of a total loss to GDP is constant.

Despite the growth of the insurance industry, most economic models, including the OLG model, do not include the insurance sector within the model and overlook the impact of insurance on the economy, such as risk allocation, financial loss compensation, assets transfers, and capital accumulation.

For the most part, previous research about the relationship between insurance and the economy has tended to focus on microanalyses or empirical studies of real data with proxy variables for insurance and the economy. Beenstock, Dickinson, and Khajuria (1986) first assess life insurance determinants by undertaking a cross-sectional analysis from a demand perspective. They also suggest the relationship between the demand for insurance and other factors, such as economic variables, life expectancy, population structure, and social security. Browne and Kim (1993) identify the dependency ratio, national income, government spending on social security, inflation, and insurance price as important determinant factors in the demand for life insurance. Ward and Zurbruegg (2000) examine the dynamic relationships between economic growth and growth in the OECD countries' insurance industry. They conducted a cointegration analysis and causality tests using the real GDP and total real premiums in each country from 1961 to 1996 and found that the causal relationships between economic growth and insurance market development can vary across countries. Heiss and Sümegi (2008) investigate the effects of both insurance investments and premiums on GDP growth in Europe. They adopt an endogenous growth model with a modified Cobb–Douglas production function as an analytical methodology and estimate the influence of technology and insurance on logarithmic forms of output. They find a significant link between insurance and economic growth, but they also emphasize the importance of the interest rate and economic development level to the insurance sector. Many studies have examined the relationship between insurance and economic growth in recent years. However, most of them are empirical studies, which mainly select proxy variables and derive results through statistical tests for certain countries over a certain period.

Since few theoretical studies that have dealt with the impact of insurance on the economy, this study provides a new framework for considering insurance within the OLG framework to analyze the impact of insurance on the economy.

The critical characteristics of the insurance model are as follows: first, the demand for insurance is determined in the individual's decision problems. The optimal choice of insurance for each agent directly increases their utility and private insurance coverage is partial. However, there is a limitation that financial losses from partial private insurance are not reflected in utility. Second, the government has chosen to provide public sector insurance for loss in which private insurance is not covered and refer to it as social insurance. Since the government has to require workers to pay taxes to provide compensation for agents, this paper also studies the impact of private or social insurance on workers' financial burdens.

The rest of this chapter discusses the following. Section 2 introduces the insurance model, including social insurance, Section 3 derives equations and initial values of exogenous variables in steady state, and Section 4 analyzes the impact of the preference for private insurance on economy.

2. Model development

This section describes the insurance model incorporated with OLG model. Households' decision problems determine the demand for private insurance, and the optimal choice of private insurance for each individual directly increases their utility. Individuals spend $I_t^i p_t^i$ on purchasing private insurance at the end of each period, where I_t^i means the level of indemnification and p_t^i means risk probability. The compensation for loss may be made by private insurance or by social insurance, assuming that the ratio of a total loss to GDP is constant. Preference for private insurance affects the individuals' insurance expenditure[2], and the percentage of loss compensated by private insurance among total loss. Each coverage ratio of private and social insurance of the total loss will affect economic agents' life−cycle behavior and the overall economy.

1) Individual decision

What is different from the modified Gertler's OLG model in Chapter 4 is that individuals spend on purchasing non−life insurance $I_t^i p_t^i$ each period and consider the present value of future cash flows of insurance spending Z_t^i in their non−financial assets.

(1) Retiree−decision problem

A retiree obtains utility from consumption C_t^r, insurance purchasing I_t^r, and leisure $1-l_t^r$ as follows:

$$\text{Max} \quad V_t^r = \left[\left\{ (C_t^r)^{\upsilon} (I_t^r)^{\nu} (1-l_t^r)^{1-\upsilon-\nu} \right\}^{\rho} + \beta\gamma (V_{t+1}^r)^{\rho} \right]^{1/\rho} \tag{5.2.1}$$

$$\text{subject to} \quad A_{t+1}^r = \tfrac{R_t}{\gamma} A_t^r + W_t^r l_t^r + E_t - C_t^r - I_t^r p_t^r \,. \tag{5.2.2}$$

The retirees' financial assets A_{t+1}^r at the beginning of period t+1 are equal to

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2 An increase in private insurance expenditure means an increase in indemnification with the risk probability unchanged.

the sum of financial assets $\frac{R_t}{\gamma}A_t^r$ held by the retiree and cash flows including labor income $W_t^r l_t^r$, social security benefits E_t, consumption C_t^r and insurance expenditures $I_t^r p_t^r$ at the end of period t.

The first−order condition for insurance, labor supply, consumption gives (Appendix (B.1.8), (B.1.9)):

$$I_t^r p_t^r = \tfrac{\nu}{\upsilon} C_t^r , \tag{5.2.3}$$

$$1 - l_t^r = \tfrac{1-\upsilon-\nu}{\upsilon} C_t^r / W_t^r . \tag{5.2.4}$$

$$C_{t+1}^r = [(\tfrac{W_t^r}{W_{t+1}^r})^{(1-\upsilon-\nu)\rho} (\tfrac{p_t^r}{p_{t+1}^r})^{\nu\rho} R_{t+1}\beta]^{\sigma} C_t^r . \tag{5.2.5}$$

Here, wage increases by x percent each year, but the risk probability is assumed to be constant each year.

On the other hand, equation (5.2.3) also can be expressed as

$$I_t^r p_t^r = \tfrac{\nu}{1-\upsilon-\nu} W_t^r (1 - l_t^r) . \tag{5.2.6}$$

Let us guess a form of consumption function as follows:

$$C_t^r = \varepsilon_t \pi_t (\tfrac{R_t}{\gamma} A_t^r + H_t^r + S_t^r - Z_t^r) , \tag{5.2.7}$$

and the retiree's consumption rate $\varepsilon_t \pi_t$ out of wealth is derived by (Appendix (B.1.27))

$$\varepsilon_t \pi_t = 1 - [(\tfrac{W_t^r}{W_{t+1}^r})^{1-\upsilon-\nu} (\tfrac{p_t^r}{p_{t+1}^r})^{\nu} R_{t+1}]^{\sigma-1} \beta^{\sigma} \gamma \tfrac{\varepsilon_t \pi_t}{\varepsilon_{t+1}\pi_{t+1}} . \tag{5.2.8}$$

In Eq. (5.2.7), non−financial assets can be evaluated as

$$H_t^r + S_t^r - Z_t^r = W_t^r l_t^r + E_t - I_t^r p_t^r + \tfrac{\gamma}{R_{t+1}} (H_{t+1}^r + S_{t+1}^r - Z_{t+1}^r) , \tag{5.2.9}$$

where H_t^r is human wealth, S_t^r is social security wealth, and Z_t^r is the total expenditure on insurance. Each asset is the present value of future cash flows from labor income, social security benefits, and insurance expenditure, respectively, and valuations by assets also hold as follows:

$$H_t^r = W_t^r l_t^r + \tfrac{\gamma}{R_{t+1}} H_{t+1}^r, \tag{5.2.10}$$

$$S_t^r = E_t + \tfrac{\gamma}{R_{t+1}} S_{t+1}^r, \tag{5.2.11}$$

$$Z_t^r = I_t^r p_t^r + \tfrac{\gamma}{R_{t+1}} Z_{t+1}^r. \tag{5.2.12}$$

(2) Worker-decision problem

As with retirees, workers choose consumption C_t^w, insurance purchasing I_t^w, and leisure $1-l_t^w$ that maximize their value function

$$V_t^w = \left\{ [(C_t^w)^{\upsilon} (I_t^w)^{\nu} (1-l_t^w)^{1-\upsilon-\nu}]^{\rho} + \beta[\omega V_{t+1}^w + (1-\omega) V_{t+1}^r]^{\rho} \right\}^{1/\rho} \tag{5.2.13}$$

subject to budget constraint

$$A_{t+1}^w = R_t A_t^w + W_t l_t^w - C_t^w - I_t^w p_t. \tag{5.2.14}$$

The worker's financial assets at the end of period t after all cash flows occur becomes either a worker's wealth or a retiree’s wealth at the beginning of period t+1. Unlike retirees, workers pay income tax per period, and they will receive social security benefits after retirement.

The first-order conditions necessary for the worker's insurance, labor supply, consumption choice are given by

$$I_t^w p_t = \tfrac{\nu}{\upsilon} C_t^w, \tag{5.2.15}$$

$$1-l_t^w = \frac{1-\upsilon-\nu}{\upsilon} C_t^w / W_t \ . \tag{5.2.16}$$

$$\omega C_{t+1}^w + (1-\omega)\chi(\varepsilon_{t+1})^{\frac{\sigma}{1-\sigma}} C_{t+1}^r = [(\frac{p_t}{p_{t+1}})^\nu (\frac{W_t}{W_{t+1}})^{(1-\upsilon-\nu)}]^{\sigma-1}(R_{t+1}\Omega_{t+1}\beta)^\sigma C_t^w \ . \tag{5.2.17}$$

On the other hand, equation (5.2.15) also can be expressed as

$$I_t^w p_t = \frac{\nu}{1-\upsilon-\nu} W_t \ (1-l_t^w) \ . \tag{5.2.18}$$

In (5.2.17), Ω_{t+1} is a risk adjustment factor defined as follows:

$$\Omega_{t+1} = \omega + (1-\omega)\frac{1}{\gamma}(\varepsilon_{t+1})^{\frac{1}{1-\sigma}}\chi \ , \tag{5.2.19}$$

where $\chi = (\frac{1}{\eta})^\nu (\frac{1}{\xi})^{(1-\upsilon-\nu)}$, $\eta = \frac{p_{t+1}^r}{p_{t+1}}$, $\xi = \frac{W_{t+1}^r}{W_{t+1}}$.

The risk adjustment factor Ω in the insurance model is different from that in the modified Gertler's model in that χ includes the relative risk probabilities in addition to the relative price of wages.

A form of consumption function can be conjectured as follows:

$$C_t^w = \pi_t (R_t A_t^w + H_t^w + S_t^w - Z_t^w) \ , \tag{5.2.20}$$

and the worker's consumption rate π_t out of wealth is derived by (Appendix (B.2.28))

$$\pi_t = 1 - [(\frac{p_t}{p_{t+1}})^\nu (\frac{W_t}{W_{t+1}})^{(1-\upsilon-\nu)} R_{t+1}\Omega_{t+1}]^{\sigma-1}\beta^\sigma \frac{\pi_t}{\pi_{t+1}} \ . \tag{5.2.21}$$

In equation (5.2.20), valuation of non-financial assets is given by (Appendix (B. 2.39)):

$$H_t^w + S_t^w - Z_t^w = W_t l_t^w - I_t^w p_t + \frac{1}{R_{t+1}}\frac{\omega}{\Omega_{t+1}}(H_{t+1}^w + S_{t+1}^w - Z_{t+1}^w) + \frac{1}{R_{t+1}}(1-\frac{\omega}{\Omega_{t+1}})\gamma(H_{t+1}^r + S_{t+1}^r - Z_{t+1}^r) \ , \tag{5.2.22}$$

where H_t^w is human wealth, S_t^w is social security wealth, and Z_t^w is the total expenditure on insurance.

In the previous Chapter 4, the consistency of valuations in human wealth and social security wealth is ensured by modifying the ambiguous transition factor in the social security assets valuation in original Gertler's model. This consistency is still ensured when the present value of future insurance expenditures Z_t^i is included in non−financial assets.

Therefore, two valuation factors contingent on the two states of employment and retirement are still in the insurance model, as in the modified model. In other words, when the worker maintains the status of employment in the next period, non−financial assets are evaluated by a factor $\frac{1}{R_{t+1}}\frac{\omega}{\Omega_{t+1}}$: the production of discounting factor $\frac{1}{R_{t+1}}$ and risk−adjusted probability $\frac{\omega}{\Omega_{t+1}}$. On the other hand, when the worker retires in the next period, non−financial assets are evaluated by a factor $\frac{1}{R_{t+1}}(1-\frac{\omega}{\Omega_{t+1}})$: the production of discounting factor $\frac{1}{R_{t+1}}$ and risk−adjusted probability $(1-\frac{\omega}{\Omega_{t+1}})$.

The following equations for valuations by asset also hold as follows:

$$H_t^w = W_t l_t^w + \frac{1}{R_{t+1}}\frac{\omega}{\Omega_{t+1}}H_{t+1}^w + \frac{1}{R_{t+1}}(1-\frac{\omega}{\Omega_{t+1}})\gamma H_{t+1}^r, \tag{4.2.36}$$

$$S_t^w = \frac{1}{R_{t+1}}\frac{\omega}{\Omega_{t+1}}S_{t+1}^w + \frac{1}{R_{t+1}}(1-\frac{\omega}{\Omega_{t+1}})\gamma S_{t+1}^r, \tag{4.2.37}$$

$$Z_t^w = I_t^w p_t + \frac{1}{R_{t+1}}\frac{\omega}{\Omega_{t+1}}Z_{t+1}^w + \frac{1}{R_{t+1}}(1-\frac{\omega}{\Omega_{t+1}})\gamma Z_{t+1}^r. \tag{5.2.23}$$

Human and social security wealth valuations of workers and retirees are the same as those in the modified Gertler’s model. Even if Z_t^w is included, the valuations in non−financial assets are still consistent.

2) Aggregate decision and the distribution of wealth

The method for deriving the aggregate equations is the same as in Gertler (1999). Because individual's consumption rate in the same group is identical, the aggregate consumption function can be easily derived by summing across the individuals' functions within same groups.

From (5.2.7) and (5.2.20), respective aggregate consumption functions of workers and retirees are as follows

$$C_t^{r\bullet} = \varepsilon_t \pi_t (R_t A_t^{r\bullet} + H_t^{r\bullet} + S_t^{r\bullet} - Z_t^{r\bullet}) = \varepsilon_t \pi_t (R_t \lambda_t A_t^{r\bullet} + H_t^{r\bullet} + S_t^{r\bullet} - Z_t^{r\bullet}), \qquad (5.2.24)$$

$$C_t^{w\bullet} = \pi_t (R_t A_t^{w\bullet} + H_t^{w\bullet} + S_t^{w\bullet} - Z_t^{w\bullet}) = \pi_t [R_t (1-\lambda_t) A_t^{\bullet} + H_t^{w\bullet} + S_t^{w\bullet} - Z_t^{w\bullet}], \qquad (5.2.25)$$

and aggregate consumption function of overall economy is as follows.

$$C_t^{\bullet} = \pi_t [\{1 + (\varepsilon_t - 1)\lambda_t\} R_t A_t^{\bullet} + (H_t^{w\bullet} + S_t^{w\bullet} - Z_t^{w\bullet}) + \varepsilon_t (H_t^{r\bullet} + S_t^{r\bullet} - Z_t^{r\bullet})]. \qquad (5.2.26)$$

Here, the aggregate equations for non−financial assets of workers and retirees are derived from (5.2.9) and (5.2.22), respectively, as follows:

$$H_t^{r\bullet} + S_t^{r\bullet} - Z_t^{r\bullet} = (W_t^r L_t^r + E_t^{\bullet} - I_t^{r\bullet} p_t^r) + \tfrac{1}{(1+n)R_{t+1}} \gamma (H_{t+1}^{r\bullet} + S_{t+1}^{r\bullet} - Z_{t+1}^{r\bullet}), \qquad (5.2.27)$$

$$\begin{aligned} & H_t^{w\bullet} + S_t^{w\bullet} - Z_t^{w\bullet} \\ & = (W_t L_t^w - T_t - I_t^{w\bullet} p_t) + \tfrac{1}{(1+n)R_{t+1}} \tfrac{\omega}{\Omega_{t+1}} (H_{t+1}^{w\bullet} + S_{t+1}^{w\bullet} - Z_{t+1}^{w\bullet}) + \tfrac{1}{(1+n)R_{t+1}} (1 - \tfrac{\omega}{\Omega_{t+1}}) \gamma (H_{t+1}^{r\bullet} + S_{t+1}^{r\bullet} - Z_{t+1}^{r\bullet}). \end{aligned}$$

(5.2.28)

Workers' share of assets $\lambda_t^w \equiv (1 - \lambda_t^r) \equiv A_t^{w\bullet} / A_t^{\bullet}$ considering the purchasing non−life insurance is given by

$$(1-\lambda_{t+1}^{r})A_{t+1}^{\bullet}=\omega[(1-\lambda_{t}^{r})R_{t}A_{t}^{\bullet}+W_{t}L_{t}^{w}-T_{t}\ -C_{t}^{w\bullet}-I_{t}^{w\bullet}p_{t}\,], \tag{5.2.29}$$

and (5.2.29) also can be expressed as

$$(1-\lambda_{t}^{r})R_{t}A_{t}^{\bullet}+W_{t}L_{t}^{w}-T_{t}\ -C_{t}^{w\bullet}-I_{t}^{w\bullet}p_{t}\ =\tfrac{(1-\lambda_{t+1}^{r})}{\omega}A_{t+1}^{\bullet}. \tag{5.2.30}$$

For retirees, the share of assets $\lambda_{t}^{r}\equiv A_{t}^{r\bullet}/A_{t}^{\bullet}$ is given by

$$\lambda_{t+1}^{r}A_{t+1}^{\bullet}=R_{t}\lambda_{t}^{r}A_{t}^{\bullet}+W_{t}^{r}L_{t}^{r}+E_{t}^{\bullet}-C_{t}^{r\bullet}-I_{t}^{r\bullet}p_{t}^{r}+(1-\omega)[(1-\lambda_{t}^{r})R_{t}A_{t}^{\bullet}+W_{t}L_{t}^{w}-T_{t}\ -C_{t}^{w\bullet}-I_{t}^{w\bullet}p_{t}\,], \tag{5.2.31}$$

and using (5.2.30), equation (5.2.31) is rewritten as

$$\lambda_{t+1}^{r}A_{t+1}^{\bullet}=R_{t}\lambda_{t}^{r}A_{t}^{\bullet}+W_{t}^{r}L_{t}^{r}+E_{t}^{\bullet}-C_{t}^{r\bullet}-I_{t}^{r\bullet}p_{t}^{r}+(1-\omega)\tfrac{(1-\lambda_{t+1}^{r})}{\omega}A_{t+1}^{\bullet} \tag{5.2.32}$$

Thus, retirees' share of assets is obtained as

$$\lambda_{t+1}^{r}=\omega(1-\varepsilon_{t}\pi_{t})R_{t}\lambda_{t}^{r}\tfrac{A_{t}^{\bullet}}{A_{t+1}^{\bullet}}+\omega[W_{t}^{r}L_{t}^{r}+E_{t}^{\bullet}-\varepsilon_{t}\pi_{t}(H_{t}^{r\bullet}+S_{t}^{r\bullet}-Z_{t}^{r\bullet})-I_{t}^{r\bullet}p_{t}^{r}]\tfrac{1}{A_{t+1}^{\bullet}}+(1-\omega)\,. \tag{5.2.33}$$

Besides, from (5.2.4) and (5.2.16), the aggregate labor supply equations are derived as follows:

$$L_{t}^{w}=N_{t}\ -\tfrac{(1-\upsilon-\nu)/\upsilon}{W_{t}}C_{t}^{w\bullet}, \tag{5.2.34}$$

$$L_{t}^{r}=N_{t}\psi-\tfrac{(1-\upsilon-\nu)/\upsilon}{\xi W_{t}}C_{t}^{r\bullet}. \tag{5.2.35}$$

3) Social insurance

The government provides social insurance for loss not covered by private insurance and we refer to social insurance expenditure as G_{t}^{s}. It is financed by the income tax and one period government bonds B_{t}. Thus, the government for each period satisfies the following conditions:

$$B_{t+1} = R_t B_t + G_t + G_t^s + E_t - T_t \,. \tag{5.2.36}$$

G_t denotes governments' general expenditure, G_t^s denotes social insurance expenditure, and E_t indicates a total amount of social security benefits for retirees. As in Gertler's basic model, the ratio of governments' general expenditure to GDP, the ratio of social security payments to GDP, and the ratio of government debt to GDP are fixed as follows:

$$G_t = \bar{g}_t Y_t \,, \quad E_t = \bar{e}_t Y_t \,, \quad B_t = \bar{b}_t Y_t \,. \tag{5.2.37}$$

On the other hand, social insurance expenditure is defined as

$$G_t^s = \text{Total loss} - I_t^w p_t - I_t^r p_t^r \,. \tag{5.2.38}$$

As shown in (5.2.38), the government subsidizes those differences through social insurance when a total loss is not fully covered by private insurance. Private insurance and social insurance are substitute relations in this framework since an increase in the demand for private insurance decreases the demand for social insurance, assuming that the ratio of a total loss to output is constant.

For example, suppose that the ratio of a total loss to output is set to 0.047[3] and this value remains constant. By increasing the preference parameter for private insurance from 0 to 0.04, the ratio of compensation to total loss by private insurance increases from 0 to 0.047 and the ratio of compensation by social insurance decreases from 0.047 to 0. That is, as the preference for

[3] Based on premium income of non-life insurance from Statistical year book 2018

private insurance increases, the coverage ratio by private insurance rather than social insurance increases for the total loss.

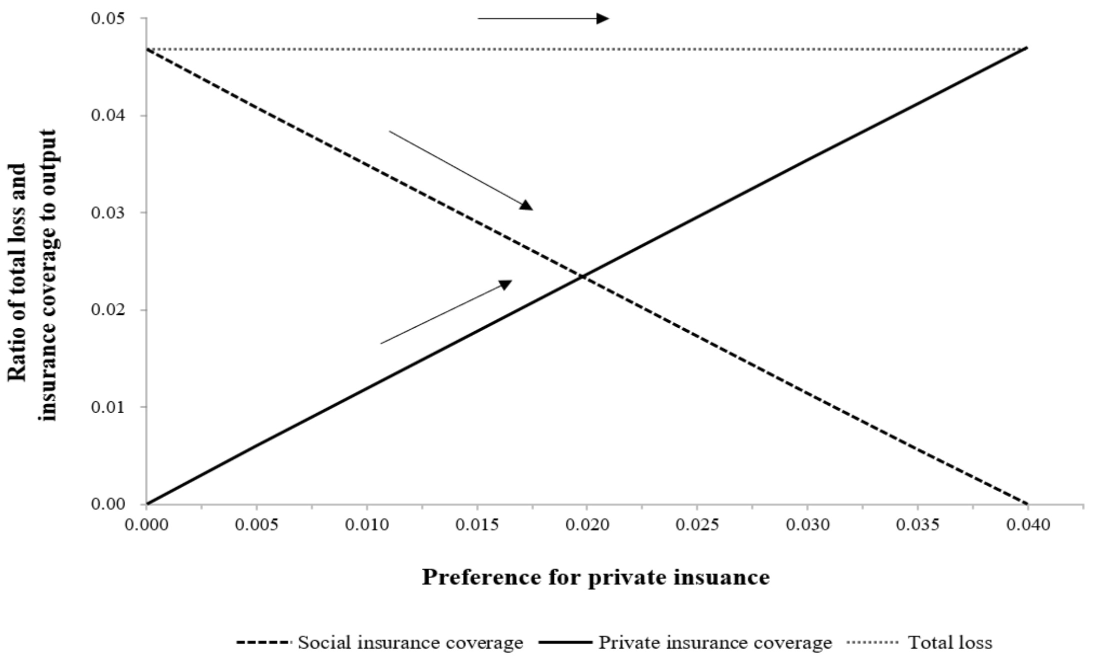

Figure 5−1. Ratio of total loss and private/social insurance coverage to output

Table 5−1. Ratio of total loss and private/social insurance coverage to output by insurance preference parameter

Insurance preference parameter	Ratio of loss covered by private insurance to output	Ratio of loss covered by social insurance to output	Ratio of total loss to output
0.000	0.000	0.047	0.047
0.005	0.006	0.041	0.047
0.010	0.012	0.035	0.047
0.015	0.018	0.029	0.047
0.020	0.024	0.023	0.047
0.025	0.030	0.017	0.047
0.030	0.035	0.011	0.047
0.035	0.041	0.006	0.047
0.040	0.047	0.000	0.047

Table 5−2. Rate of loss covered by private/social insurance to total loss by insurance preference parameter

Insurance preference parameter	% of loss covered by private insurance to total loss	% of loss covered by social insurance to total loss	Total loss
0.000	0.00%	100.0%	100%
0.005	12.7%	87.3%	100%
0.010	25.4%	74.6%	100%
0.015	38.1%	61.9%	100%
0.020	50.6%	49.4%	100%
0.025	63.2%	36.8%	100%
0.030	75.7%	24.3%	100%
0.035	88.1%	11.9%	100%
0.040	100.0%	0.00%	100%

As in Gertler's basic model, financial wealth equals the sum of capital and government debt,

$$A_t = K_t + B_t , \tag{5.2.39}$$

and the capital intensity, vehicle for saving, in the insurance model, evolves as

$$K_{t+1} = Y_t - C_t - G_t - G_t^s - I_t^w p_t - I_t^r p_t^r + (1-\delta)K_t . \tag{5.2.40}$$

3. Equations and initial values in steady state

1) Steady−state equations

The methods for deriving the steady−state equations follow those suggested in Gertler (1999) and are described in detail in Chapter 2. Because all quantity variables grow at the effective growth rate $(1+x)(1+n)$, they are normalized by dividing them into output in steady state, for convenience.

From equation (5.2.40), the capital stock to output k can be derived as

$$\left[(1+x)(1+n)-1+\delta\right]k = 1-c-g-g^{s}-\iota^{w}p-\iota^{r}p^{r}. \qquad (5.2.41)$$

In equation (5.2.41), the left side means investment and the right side means saving. The steady-state equation of total consumption is given by

$$c=\pi\left[\left\{1+(\varepsilon-1)\lambda^{r}\right\}R(k+b)+(h^{w}+s^{w}-z^{w})+\varepsilon(h^{r}+s^{r}-z^{r})\right], \qquad (5.2.42)$$

and workers' and retirees' consumption functions in steady state are as follows, respectively,

$$c^{r}=\varepsilon\pi[\lambda^{r}R(k+b)+h^{r}+s^{r}-z^{r}] \qquad (5.2.43)$$

$$c^{w}=\pi[(1-\lambda^{r})R(k+b)+h^{w}+s^{w}-z^{w}]. \qquad (5.2.44)$$

Substituting (5.2.42) into (5.2.41), then,

$$\left[(1+x)(1+n)-1+\delta\right]k$$
$$=1-\pi\left\{[1+(\varepsilon-1)\lambda^{r}]R(k+b)+(h^{w}+s^{w}-z^{w})+\varepsilon(h^{r}+s^{r}-z^{r})\right\}-g-g^{s}-\iota^{w}p-\iota^{r}p^{r}.$$

(5.2.45)

Human wealth for workers and retirees in steady state are as follows:

$$h^{r}=\alpha\tfrac{L-L^{w}}{L}/(1-\tfrac{(1+x)}{R}\gamma), \qquad (5.2.46)$$

$$h^{w}=[\alpha\tfrac{L^{w}}{L}-\tau+\tfrac{(1+x)}{R}(1-\tfrac{\omega}{\Omega})\gamma h^{r}]/[1-\tfrac{1}{R}\tfrac{\omega(1+x)}{\Omega}], \qquad (5.2.47)$$

where $$\tfrac{L^{w}}{L}=\tfrac{N}{L}-\tfrac{(1-\upsilon-\nu)}{\alpha\upsilon}c^{w}, \qquad (5.2.48)$$

$$\tfrac{L}{N}=(1+\xi\psi)[1+\tfrac{(1-\upsilon-\nu)}{\alpha\upsilon}c]^{-1}. \qquad (5.2.49)$$

Steady state equations of social security wealth for workers and retirees are given by

$$s^r = e/(1-\tfrac{(1+x)}{R}\gamma), \qquad (5.2.50)$$

$$s^w = \tfrac{(1+x)}{R}(1-\tfrac{\omega}{\Omega})\gamma s^r / (1-\tfrac{(1+x)}{R}\tfrac{\omega}{\Omega}). \qquad (5.2.51)$$

The equation for the share of assets of retirees in steady state is given by

$$\lambda^r = \omega R(1-\varepsilon\pi)\lambda^r \tfrac{1}{(1+x)(1+n)} + \omega[\alpha \tfrac{L-L^w}{L} + e - \varepsilon\pi(h^r + s^r - z^r) - \iota^r p^r]\tfrac{1}{(1+x)(1+n)(k+b)} + (1-\omega) \qquad (5.2.52)$$

$$\lambda^w = 1-\lambda^r \qquad (5.2.53)$$

In steady state, gross return is

$$R = (1-\alpha)k^{-1} + (1-\delta), \qquad (5.2.54)$$

and substituting (5.2.45) into (5.2.54), then equation (5.2.54) is rewritten as

$$R = \frac{1-[(1+x)(1+n)-1+\delta]k-\iota^w p-\iota^r p^r-\pi[(h^w+s^w-z^w)+\varepsilon(h^r+s^r-z^r)]-g-g^s}{\pi[1+(\varepsilon-1)\lambda^r](k+b)}. \qquad (5.2.55)$$

Risk adjustment factor and the propensity to consume of workers and retirees are respectively given by

$$\Omega = [\omega + (1-\omega)\tfrac{1}{\gamma}(\varepsilon_{t+1})^{\frac{1}{1-\sigma}}\chi], \qquad (5.2.56)$$

$$\pi = 1-[(\tfrac{1}{1+x})^{(1-\upsilon-\nu)}R\Omega]^{\sigma-1}\beta^{\sigma}, \qquad (5.2.57)$$

$$\varepsilon\pi = 1-[(\tfrac{1}{1+x})^{(1-\upsilon-\nu)}R]^{\sigma-1}\beta^{\sigma}\gamma. \qquad (5.2.58)$$

Finally, the total tax-to-output ratio is as follows:

$$\tau = [R-(1+x)(1+n)]b + g + g^s + e. \qquad (5.2.59)$$

2) Initial steady-state values

In the insurance model, in addition to the parameters presented in Gertler's

(1999) paper, the following parameters are added: preference parameter for private non−life insurance ν, the relative value of risk probability η, which is the ratio of the risk probability of a retiree to the risk probability of a worker. Here, the government has no provision of social insurance by assuming that the total loss is fully covered by private insurance when the insurance preference parameter value is 0.04.

Table 5−3 shows the value and description of parameters, and Table 5−4 shows the steady−state values of basic exogenous variables.

Table 5−3. Description and value of parameters

Parameter	Description	Value
n	Workforce growth rate	0.01
ω	Probability of remaining in the labor force	0.977
γ	Probability of surviving of a retiree	0.92
υ	Preference parameter for consumption	0.4
ν	Preference parameter for private nonlife insurance	0.04
$1-\upsilon-\nu$	Preference parameter for leisure	0.56
β	Subjective discount rate	1.0
ρ	Curvature parameter	−3
σ	Intertemporal elasticity of substitution	0.25
ξ	Productivity of a unit of labor supplied by a retiree relative to a worker	0.6
η	Ratio of the risk probability of a retiree to the risk probability of a worker	3
α	Labor income share ratio	0.667
δ	Capital depreciation rate	0.1
x	Growth rate of technology	0.01
b	Government debt to output	0.25
g	Government consumption to output expenditure	0.20
e	Social security payments to output	0.02
p	Risk probability of a worker	0.01
p^r	Risk probability of a retiree	0.03

Table 5-4. Initial steady-state values of basic exogenous variables

k	Capital stock to output	2.353
K / XL	Capital stock per unit of effective labor	3.609
R	Gross return on capital	1.042
π	The propensity to consume out of wealth	0.058
$\varepsilon\pi$		0.104
Ω	Risk adjustment factor	1.046
h^w	Ratio of human wealth to output	4.527
h^r		0.298
s^w	Ratio of social security wealth to output	0.115
s^r		0.185
z^w	Ratio of present value of total insurance expenditure to output	0.449
z^r		0.095
λ^w	Share of financial assets	0.778
λ^r		0.222
τ	Ratio of total tax to output	0.225
c^w	Ratio of consumption to output	0.367
c^r		0.103
$c = c^w + c^r$		0.470
$\iota^w p$	Ratio of insurance expenditure to output	0.037
$\iota^r p^r$		0.010
$\iota p = \iota^w p + \iota^r p^r$		0.047
L^w / N	Labor supply as a fraction of total time endowment	0.552
$L^r / N\psi$		0.182

Notes: 1) superscript w denotes workers, and r denotes retirees.

4. Impacts of expanding coverage through private insurance

This section examines the effects of an increase in the proportion of private insurance on the economic agents' behavior and the macroeconomy, i.e., when the proportion of claims payment to the total loss by private insurance increases.

As previous mentioned, when the ratio of total loss amounts to output is constant, increasing the preference parameter for private insurance increases the rate of loss covered by private insurance of the total loss. When that happens, the rate of loss covered by social insurance decreases. (see Table 5−1 and Table 5−2). Varying the ratio of private insurance coverage to a total loss from 0 to 1 increases capital stock per unit of effective labor from 3.48 to 3.61, a change of 3.79%, and reduces the gross return on capital from 1.045 to 1.042, a 0.34% change.

Table 5−5. Effects of the ratio of private insurance coverage to total loss on capital stock and gross return

The ratio of private insurance coverage to total loss	Capital stock per unit of effective labor (K/XL)	Gross return on capital stock (R)
0.000	3.478	1.0452
0.127	3.493	1.0448
0.254	3.511	1.0443
0.381	3.527	1.0439
0.506	3.545	1.0434
0.632	3.561	1.0429
0.757	3.577	1.0425
0.881	3.594	1.0421
1.000	3.609	1.0417
	△0.132	▽0.004
	△3.79%	▽0.34%

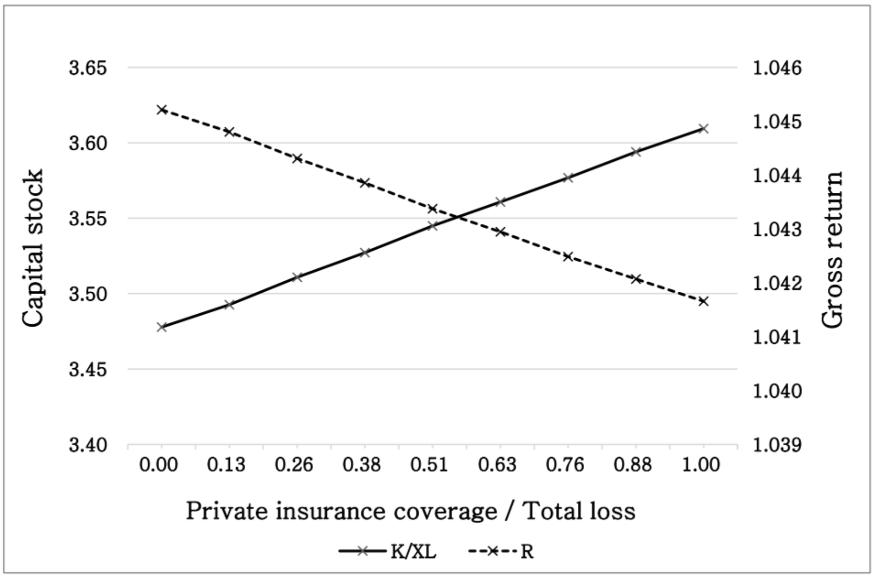

Figure 5-2. Effects of the ratio of private insurance coverage to total loss on capital stock and gross return

Table 5-6. Effects of the ratio of private insurance coverage to total loss on the tax burden

The ratio of private insurance coverage to total loss	Total tax to output ($\tau = T/Y$)	Total tax to labor income($\tau/\alpha\frac{L^w}{L}$)
0.000	0.273	0.425
0.127	0.267	0.416
0.254	0.261	0.408
0.381	0.255	0.399
0.506	0.249	0.390
0.632	0.243	0.381
0.757	0.237	0.372
0.881	0.231	0.364
1.000	0.225	0.355
	▽0.048	▽0.07

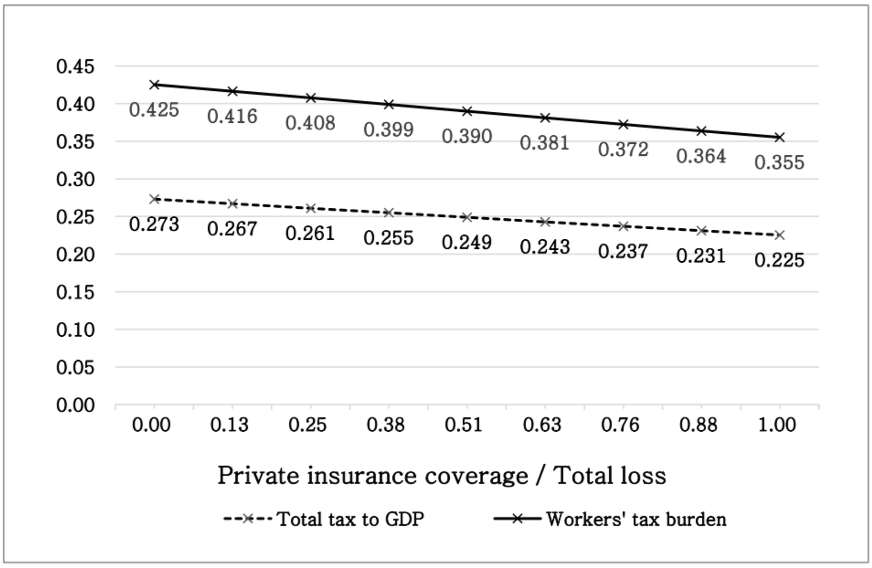

Figure 5–3. Effects of the ratio of private insurance coverage to total loss on the tax burden

As shown in Table 5–6 and Figure 5–3, the percentage of total tax to GDP from 27.3% to 22.5% decreases by 4.8%p, as the ratio of private insurance coverage to a total loss increases from 0 to 1. Based on wage income, the percentage of total tax to labor income for workers decreases from 42.5% to 35.5%, by 7%p. Conversely, if the ratio of social insurance coverage to total loss increases from 0 to 1, the percentage of total tax to GDP increases by 4.8%p, and workers' tax burden to labor income increases by 7%p.

There is only a difference between paying tax or paying contribution for workers, but there is no difference in that they bear their own loss whether loss is compensated by private insurance or by social insurance. However, as the relative proportion of social insurance against total losses increases, the income tax burden on workers increases even more because retirees do not pay taxes, and workers act as a source of funding to provide social insurance services to retirees through tax payments. On the other hand, increasing the preference parameter for insurance under the fixed preference parameter for consumption, reduces the time spent on leisure relatively, thus increasing the labor supply for workers and retirees.

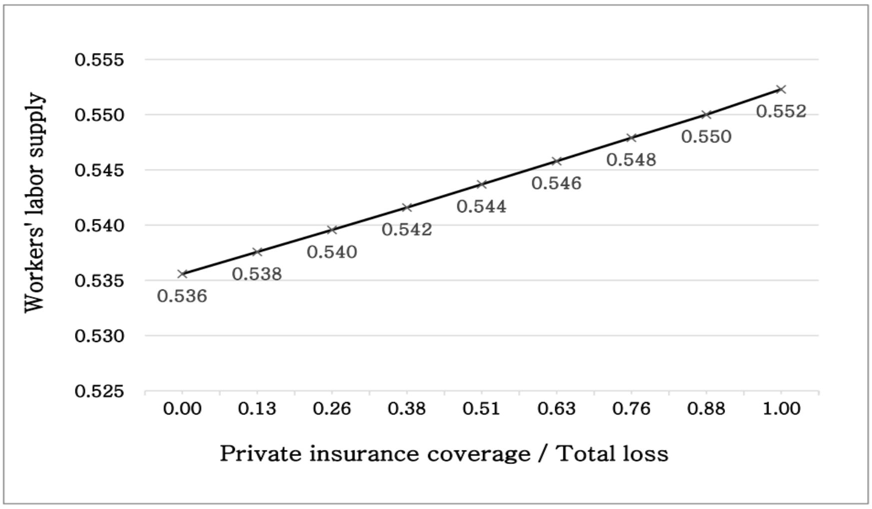

Figure 5-4. Effects of the ratio of private insurance coverage to total loss on workers' labor supply

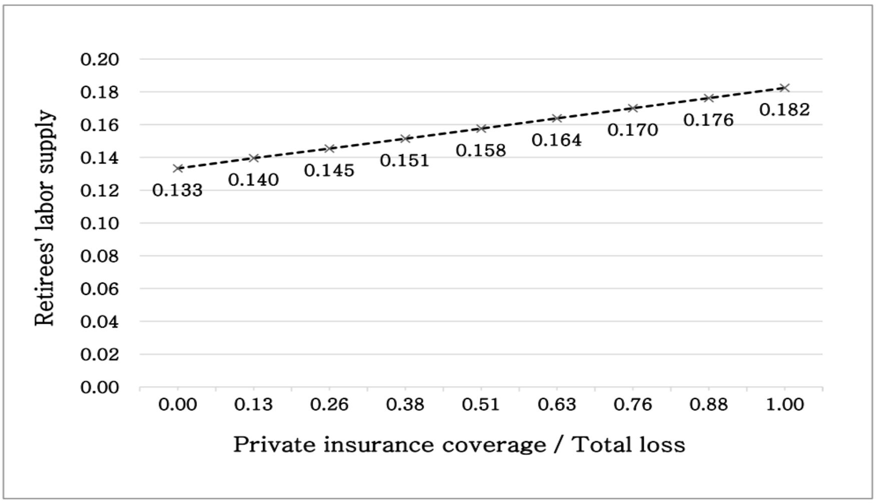

Figure 5-5. Effects of the ratio of private insurance coverage to total loss on retirees' labor supply

5. Sub−conclusion

This chapter proposes a new type of OLG model that incorporates insurance, which is meaningful in that few economic models could analyze the interactions between the economy and insurance. This insurance model can also recognize the loss from a particular financial event related to health, travel, automobiles, accidents, which were not recognized by existing economic models.

Individual's insurance purchases directly increase their utility, but there is a limitation that financial losses that are not fully covered by partial private insurance are not reflected in utility. Therefore, this model addresses such financial losses by introducing social insurance. In this model, private insurance and social insurance are substitute relations assuming that the ratio of a total loss to GDP is constant. However, it is controversial whether the relationship between private and social insurance is substitutes or complements because it can vary depending on the country's situation, such as the insurance system, policy, economy, culture, and many other factors. Therefore, a more explicit establishment of the relationship between private and social insurance and reflection in the model will contribute to the more sophisticated model.

Chapter 6.　Life Insurance Model

1. Introduction

This chapter proposes a life insurance model, an extension of the previously introduced insurance model. The life insurance model has several assumptions different from the previously mentioned insurance model regarding population composition and transition probabilities. In addition to workers and retirees, dependents are newly added as economic agents, and the survival rate is imposed on workers. Thus, it allows a dependent on receiving inheritance and death benefits from life insurance in the event of a worker's death. The risk adjustment factor is modified by including the risk probability of workers' death.

As seen in the literature review in the previous chapter, theoretical work has been relatively little done to make an economic model where insurance is incorporated. Thus, this study suggests a new type of OLG model that modified Gertler's (1999) model to include non−life and life insurance.

The structure of this chapter is as follows. Section 2 introduces the basic assumptions of our model: population dynamics, insurance sector, preferences. Section 3 deals with the individual choice to derive the behavior of individual consumption. Section 4 deals with the aggregated choice and derives steady−state equations and steady−state results for the economy. The distribution of wealth among groups and production assumptions are discussed.

2. Basic assumptions

1) Population dynamics

This model contains three types of economic agents: workers, retirees, and dependents. Each individual is born a worker, and a surviving worker is transferred to continuing to be a worker or to be a retiree, and a worker’s death causes a new dependent to emerge in the next period.

Let γ_i be a probability of survival, where subscripts $i = w, r, d$ indicate whether the individual is a worker (w), a retiree (r), or a dependent (d) and let ω be a probability of remaining in the labor force. A worker can then maintain the worker’s status with a probability of $\gamma_w \omega$, retire with a probability of $\gamma_w(1-\omega)$, or die with a probability of $1-\gamma_w$. It is possible for a retiree and a dependent on surviving with a probability of γ_r and γ_d, or die with a probability of $1-\gamma_r$ and $1-\gamma_d$, respectively. These assumptions of the transition probabilities are summed up in Table 6-1 and Figure 6-1. The transition probability is independent of the period or age of employment, which makes the aggregation easier. Thus, representative workers' expected time to remain in the labor force is $1/(1-\gamma_w \omega)$, and the remaining life expectancy of representative retirees and dependents is $1/(1-\gamma_r)$ and $1/(1-\gamma_d)$, respectively.

Table 6-1. Assumptions of the transition probabilities

Type of agent	Probability of survival	Probability of remaining in the labor force	Probability of exit from the labor force
Worker	γ_w	$\gamma_w\omega$	$1-\gamma_w\omega$
Dependent	γ_d	γ_d	$1-\gamma_d$
Retiree	γ_r	γ_r	$1-\gamma_r$

It is assumed that the number of workers N_t increases at a constant rate n each year and that new workers $(1-\gamma_w\omega+n)N_t$ are born in t+1. Then, the composition of workers in t+1 is as follows.

$$N_{t+1}=(1+n)N_t=\gamma_w\omega N_t+(1-\gamma_w\omega+n)N_t\,. \tag{6.2.1}$$

The ratio of retirees to workers ψ_r can be derived using equation (6.2.2). In the stationary equilibrium, the ratio is fixed. Thus, the number of retirees at time t is $\psi_r N_t$, and that number grows at the same rate as the work force, n.

$$\sum_{s=1}^{\infty}\gamma_r^{s-1}\gamma_w(1-\omega)N_{t-s}=\sum_{s=1}^{\infty}\gamma_r^{s-1}\gamma_w(1-\omega)\tfrac{N_t}{(1+n)^s}=\tfrac{\gamma_w(1-\omega)N_t}{1+n}\sum_{s=1}^{\infty}(\tfrac{\gamma_r}{1+n})^{s-1}=\tfrac{\gamma_w(1-\omega)N_t}{1+n-\gamma_r}\,, \tag{6.2.2}$$

$$\psi_r=\tfrac{\gamma_w(1-\omega)}{1+n-\gamma_r}\,. \tag{6.2.3}$$

Similarly, the ratio of dependents to workers is ψ_d, and the number of dependents at time t is $\psi_d N_t$.

$$\psi_d=\tfrac{1-\gamma_w}{1+n-\gamma_d}\,. \tag{6.2.4}$$

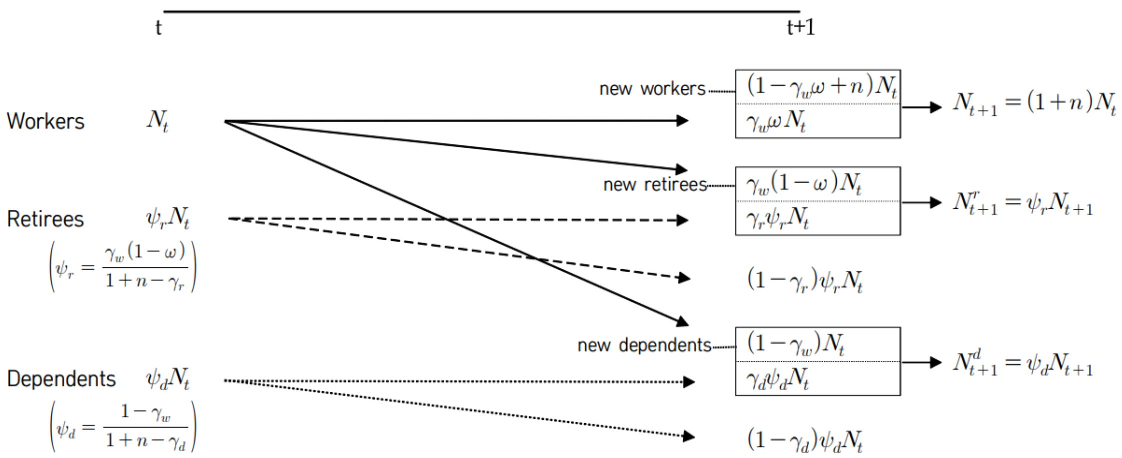

Figure 6−1. Population dynamics by transition probability

2) Insurance sector

(1) Actuarial note

Retirees and dependents eliminate death uncertainty by purchasing an actuarial note. Retirees who survive with the probability of γ_r until the next period receive the gross return R/γ_r, and those who die with the probability of $1-\gamma_r$ receive nothing. In other words, each surviving retiree receives a return that is proportionate to their initial wealth. Similarly, the gross return on wealth for a surviving dependent is R/γ_d, and when they die, they receive nothing.

(2) Non−life and life insurance

In this model, economic agents take out insurance and obtain utility from insurance in addition to consumption and leisure in their decision problems. It is assumed that only a worker can be insured for life insurance, but that all agents can be insured for non–life insurance. Life insurance is limited to term insurance, a type of life insurance that guarantees a specific period and provides death benefits if the insured dies during the period specified.

Workers spend $I_t^{w2} p_t^{w2}$ on purchasing life insurance at the end of each period, where I_t^{w2} means indemnification of life insurance and p_t^{w2} means risk probability. The risk probability p_t^{w2} in life insurance also means the worker's mortality rate $1-\gamma_w$. A dependent receives inheritance and death benefits from life insurance in the event of a worker's death.

Non–life insurance is a product that takes risks from particular financial losses related to automobiles, health, travel, home, accidents, commerce, fire, property, crops, etc. p_t^{w1}, p_t^r, p_t^d and I_t^{w1}, I_t^r, I_t^d denote the risk probability and indemnification of non–life insurance, for a worker, a retiree, a dependent, respectively. Thus, each individual by group spend $I_t^{w1} p_t^{w1}$, $I_t^r p_t^r$, $I_t^d p_t^d$ on the non–life insurance at the end of the period, respectively.

Table 6–2. Assumptions of insurance

Type of agents	Type of insurance	Risk probability	Indemnification	Insurance expenditures
Worker (w)	Non–life	p_t^{w1}	I_t^{w1}	$I_t^{w1} p_t^{w1}$
	Life	p_t^{w2}	I_t^{w2}	$I_t^{w2} p_t^{w2}$
Retiree (r)	Non–life	p_t^r	I_t^r	$I_t^r p_t^r$
Dependent (d)	Non–life	p_t^d	I_t^d	$I_t^d p_t^d$

3) Preferences

The non−expected utility function proposed by Farmer (1990) is used in this model. By imposing the survival rate on workers and by adding dependents to economic agents, the value function V_t^i can be expressed as follows:

$$V_t^i = \left\{ [(C_t^i)^{\upsilon} (I_t^i)^{\nu} (1-l_t^i)^{1-\upsilon-\nu}]^{\rho} + \beta\gamma_i [E_t(V_{t+1} \mid i)]^{\rho} \right\}^{1/\rho}, \tag{6.2.5}$$

where the superscript $i = w, r, d$ indicates whether the individual is a worker (w), a retiree (r), or a dependent (d).

$E_t(V_{t+1} \mid i)$ is the expectation of the value function in the next period conditional on the person being type i at time t and being alive at t + 1.

$$E_t(V_{t+1} \mid w) = \gamma_w \omega V_{t+1}^w + \gamma_w (1-\omega) V_{t+1}^r, \tag{6.2.6}$$

$$E_t(V_{t+1} \mid r) = V_{t+1}^r \quad \text{and} \quad E_t(V_{t+1} \mid d) = V_{t+1}^d.$$

Individuals consume three goods, such as consumption C_t, leisure $1-l_t$, and insurance I_t, but what is different from the insurance model in Chapter 5 is that the worker's insurance includes life insurance. In the previous chapters, the worker's subjective discount factor is β, but by imposing survival rates on the worker in this model, all agents use an effective subjective discount factor $\beta\gamma_i$, considering the probability of survival for each period. Meanwhile, ρ indicates the curvature parameter to smooth the trade−off between consumption and savings, and $\sigma = 1/(1-\rho)$ is the intertemporal elasticity of substitution.

3. Individual decision .

All agents supply labor, and the labor productivity of dependents and retirees is lower than that of workers. Let $\xi_r \in (0,1)$ and $\xi_d \in (0,1)$ be the productivity of labor supplied by a retiree and a dependent relative to a worker, and then the wage per unit of time is as follows: $W^w = W$, $W^r = \xi_r W$, $W^r = \xi_d W$.

The government implements fiscal and social security policies. The government pays social security benefits to retirees and dependents, and imposes taxes only on workers to raise policy funds.

Each worker determines the total insurance expenditure for both non−life and life insurance from the optimal choice. For workers, the imposition of the survival rate on each period affects the risk adjustment factor, subjective discount rate, and asset valuations. Furthermore, a dependent's initial financial assets include the bequest and insurance benefits from a worker's deaths. The life insurance model also can capture the transfer of assets from workers to retirees in aggregate decision step.

Since retirees and dependents purchase only non−life insurance and there is no difference in individual decision problems between retirees and dependents. Therefore, the solutions to decision problems of retirees and dependents in this model are the same as retirees' decision problems in the non−life insurance model of Chapter 5.

1) Retiree and dependent decision problem

The value function of retirees and dependents are

$$V_t^j = \left\{[(C_t^j)^{\upsilon}(I_t^j)^{\nu}(1-l_t^j)^{1-\upsilon-\nu}]^{\rho} + \beta\gamma_j (V_{t+1}^j)^{\rho}\right\}^{1/\rho}, \qquad (6.3.1)$$

where j=r (retiree), d (dependent).

The budget constraints are

$$A_{t+1}^j = \tfrac{R_t}{\gamma_j} A_t^j + W_t^j l_t^j + E_t^j - C_t^j - I_t^j p_t^j, \qquad (6.3.2)$$

which is the evolution of financial assets where R_t / γ_j is the gross return, $W_t^j l_t^j$ is labor income, E_t^j is social security benefits, and C_t^j is consumption, $I_t^j p_t^j$ is the expenditure on non-life insurance at the end of the period.

The first-order conditions for insurance, labor supply of the retiree and dependent yields: (Appendix (C.1.8) and (C.1.9)):

$$I_t^j p_t^j = \tfrac{\nu}{\upsilon} C_t^j, \qquad (6.3.3)$$

$$1 - l_t^j = \tfrac{1-\upsilon-\nu}{\upsilon} C_t^j / W_t^j. \qquad (6.3.4)$$

where l_t^j is labor supplied by a retiree or a dependent.

Another expression of Eq. (6.3.3) is

$$I_t^j p_t^j = \tfrac{\nu}{(1-\upsilon-\nu)} W_t^j (1 - l_t^j). \qquad (6.3.5)$$

The consumption Euler equation for the retiree and the dependent yields (see Appendix (C.1.19)):

$$C_{t+1}^j = [(\tfrac{W_t^j}{W_{t+1}^j})^{(1-\upsilon-\nu)\rho} (\tfrac{p_t^j}{p_{t+1}^j})^{\nu\rho} R_{t+1}\beta]^{\sigma} C_t^j. \qquad (6.3.6)$$

A conjecture form of consumption function is

$$C_t^j = \varepsilon_t^j \pi_t (\tfrac{R_t}{\gamma_j} A_t^j + H_t^j + S_t^j - Z_t^j), \tag{6.3.7}$$

where $\varepsilon_t^j \pi_t$ is the retiree's or the dependent's consumption rate of wealth. $\varepsilon_t^j \pi_t$ is derived by (see Appendix (C.1.27))

$$\varepsilon_t^j \pi_t = 1 - [(\tfrac{W_t^j}{W_{t+1}^j})^{1-\upsilon-\nu} (\tfrac{p_t^j}{p_{t+1}^j})^{\nu} R_{t+1}]^{\sigma-1} \beta^{\sigma} \gamma_j \tfrac{\varepsilon_t^j \pi_t}{\varepsilon_{t+1}^j \pi_{t+1}}. \tag{6.3.8}$$

Valuation of non-financial assets of retirees or dependents are

$$H_t^j + S_t^j - Z_t^j = W_t^j l_t^j + E_t^j - I_t^j p_t^j + \tfrac{\gamma_j}{R_{t+1}} (H_{t+1}^j + S_{t+1}^j - Z_{t+1}^j) \tag{6.3.9}$$

where H_t^j is human wealth, S_t^j is social security wealth, and Z_t^j is the present value of future expenditure on insurance, and cash-flows generated at t-point include labor income, social security benefits, and insurance expenditure.

The following equations for valuations by asset also hold (Appendix (C.1.36)).

$$H_t^j = W_t^j l_t^j + \tfrac{\gamma_j}{R_{t+1}} H_{t+1}^j, \tag{6.3.10}$$

$$S_t^j = E_t + \tfrac{\gamma_j}{R_{t+1}} S_{t+1}^j, \tag{6.3.11}$$

$$Z_t^j = I_t^j p_t^j + \tfrac{\gamma_j}{R_{t+1}} Z_{t+1}^j. \tag{6.3.12}$$

2) Worker decision problem

A worker chooses consumption C_t^w, leisure $1-l_t^w$, and insurance I_t^w, and faces a surviving probability of γ^w in each period.

The value function of workers are

$$V_t^w = \left\{[(C_t^w)^{\upsilon}(I_t^w)^{\nu}(1-l_t^w)^{1-\upsilon-\nu}]^{\rho} + \beta\gamma_w[\omega V_{t+1}^w + (1-\omega)V_{t+1}^r]^{\rho}\right\}^{1/\rho}, \tag{6.3.13}$$

subject to budget constraint

$$A_{t+1}^w = R_t A_t^w + W_t\, l_t^w - C_t^w - I_t^w p_t\ , \tag{6.3.14}$$

where p_t is risk probability and $I_t^w p_t = I_t^{w1} p_t^{w1} + I_t^{w2} p_t^{w2}$ ($I_t^{w1} p_t^{w1}$: expenditure on non−life insurance, $I_t^{w2} p_t^{w2}$: expenditure on life insurance). A_t^w indicates financial assets at t, R_t is the gross return, $W_t\, l_t^w$ is after−tax labor income, $I_t^w p_t$ is insurance expenditure on non−life and life insurance, at the end of the period. Workers receive social security benefits after retirement and pays labor income tax. The worker's financial wealth at the end of time t becomes either a worker’s wealth or a retiree's wealth at the beginning of time t+1.

From the first−order conditions, the following insurance and labor equations for the worker are derived as(see Appendix (C.2.8), (C.2.9)):

$$I_t^w p_t = \tfrac{\nu}{\upsilon} C_t^w\ , \tag{6.3.15}$$

$$1 - l_t^w = \tfrac{1-\upsilon-\nu}{\upsilon} C_t^w / W_t\ . \tag{6.3.16}$$

Eq.(6.3.15) can be also expressed as

$$I_t^w p_t = \tfrac{\nu}{(1-\upsilon-\nu)} W_t\ (1 - l_t^w)\,. \tag{6.3.17}$$

The first−order condition for the worker's consumption choice yields (see Appendix (C.2.21)):

$$\omega C_{t+1}^w + (1-\omega)\chi_r(\varepsilon_{t+1})^{\frac{\sigma}{1-\sigma}} C_{t+1}^r = [(\tfrac{p_t}{p_{t+1}})^{\nu}(\tfrac{W_t}{W_{t+1}})^{(1-\upsilon-\nu)}]^{\sigma-1}(R_{t+1}\Omega_{t+1}\beta)^{\sigma} C_t^w\,. \tag{6.3.18}$$

In (6.3.6), Ω_{t+1} is a risk adjustment factor given by

$$\Omega_{t+1} = \gamma_w[\omega + (1-\omega)\tfrac{1}{\gamma_r}(\varepsilon^r_{t+1})^{\frac{1}{1-\sigma}}\chi_r], \tag{6.3.19}$$

where $\chi_r = (\frac{1}{\eta_r})^{\nu}(\frac{1}{\xi_r})^{(1-\upsilon-\nu)}$, $\eta_r = \frac{p^r_{t+1}}{p_{t+1}}$, $\xi_r = \frac{W^r_{t+1}}{W_{t+1}}$

The life insurance model includes γ_w in the risk adjustment factor by modifying the worker's transition probabilities (compare ④ to ③ in Table 6−3).

Table 6−3. Definition of risk adjustment factor by models

① Gertler (1999)	$\Omega^G_{t+1} = \omega + (1-\omega)(\varepsilon_{t+1})^{\frac{1}{1-\sigma}}\chi$ Where $\xi = \frac{W^r_{t+1}}{W_{t+1}}$, $\chi = (\frac{1}{\xi})^{(1-\upsilon)}$
② Modified model	$\Omega_{t+1} = \omega + (1-\omega)\frac{1}{\gamma}(\varepsilon_{t+1})^{\frac{1}{1-\sigma}}\chi$ Where $\xi = \frac{W^r_{t+1}}{W_{t+1}}$, $\chi = (\frac{1}{\xi})^{(1-\upsilon)}$
③ Insurance model	$\Omega_{t+1} = \omega + (1-\omega)\frac{1}{\gamma}(\varepsilon_{t+1})^{\frac{1}{1-\sigma}}\chi$ Where $\xi = \frac{W^r_{t+1}}{W_{t+1}}$, $\eta = \frac{p^r_{t+1}}{p_{t+1}}$, $\chi = (\frac{1}{\eta})^{\nu}(\frac{1}{\xi})^{(1-\upsilon-\nu)}$
④ Life insurance model	$\Omega_{t+1} = \gamma_w[\omega + (1-\omega)\frac{1}{\gamma_r}(\varepsilon^r_{t+1})^{\frac{1}{1-\sigma}}\chi_r]$ Where $\xi_r = \frac{W^r_{t+1}}{W_{t+1}}$, $\eta_r = \frac{p^r_{t+1}}{p_{t+1}}$ $\chi_r = (\frac{1}{\eta_r})^{\nu}(\frac{1}{\xi_r})^{(1-\upsilon-\nu)}$

Notes: 1) In the life insurance model, r, which represents retirees, is used as a subscript or a superscript of γ, ε, ξ, η, χ to distinguish it from dependents. 2) The risk probability of workers p_{t+1} in the life insurance model is a weighted average of the risk probability of non−life insurance and the risk probability of life insurance.

Let us guess a form of consumption function as follows:

$$C^w_t = \pi_t(R_t A^w_t + H^w_t + S^w_t - Z^w_t), \tag{6.3.20}$$

where π_t is the worker's propensity to consume out of wealth. π_t is derived by (see Appendix (C.2.27))

$$\pi_t = 1 - [(\tfrac{p_t}{p_{t+1}})^{\nu}(\tfrac{W_t}{W_{t+1}})^{(1-\upsilon-\nu)}R_{t+1}\Omega_{t+1}]^{\sigma-1}\beta^{\sigma}\gamma_w\tfrac{\pi_t}{\pi_{t+1}}. \tag{6.3.21}$$

On the other hand, the valuation of non−financial assets for workers should consider both future cash−flows depending on the two possibilities of maintaining a worker's status and retiring by transition probabilities. Therefore, the worker’s non−financial assets $H_t^w + S_t^w - Z_t^w$ are evaluated by

$$H_t^w + S_t^w - Z_t^w = W_t l_t^w - I_t^w p_t + \tfrac{1}{R_{t+1}} \tfrac{\gamma_w \omega}{\Omega_{t+1}} (H_{t+1}^w + S_{t+1}^w - Z_{t+1}^w) + \tfrac{1}{R_{t+1}} (1 - \tfrac{\gamma_w \omega}{\Omega_{t+1}}) \gamma_r (H_{t+1}^r + S_{t+1}^r - Z_{t+1}^r), \quad (6.3.22)$$

where H_t^w is human wealth, S_t^w is social security wealth, and Z_t^w is the present value of future insurance expenditure.

Workers' non−financial assets are evaluated by two state−contingent factors, which are the production of discounting factor and risk−adjusted probabilities, as shown in (6.3.22). When the worker remains in the labor force in the next period, $\frac{1}{R_{t+1}} \frac{\gamma_w \omega}{\Omega_{t+1}}$ is used, and when the worker goes from work to retirement in the next period, $\frac{1}{R_{t+1}} (1 - \frac{\gamma_w \omega}{\Omega_{t+1}})$ is used. Here, Ω plays a role in adjusting transition probabilities $\gamma_w \omega$ in valuations.

The following equations for valuations by asset also hold.

$$H_t^w = W_t l_t^w + \tfrac{1}{R_{t+1}} \tfrac{\gamma_w \omega}{\Omega_{t+1}} H_{t+1}^w + \tfrac{1}{R_{t+1}} (1 - \tfrac{\gamma_w \omega}{\Omega_{t+1}}) \gamma_r H_{t+1}^r \quad (6.3.23)$$

$$S_t^w = \tfrac{1}{R_{t+1}} \tfrac{\gamma_w \omega}{\Omega_{t+1}} S_{t+1}^w + \tfrac{1}{R_{t+1}} (1 - \tfrac{\gamma_w \omega}{\Omega_{t+1}}) \gamma_r S_{t+1}^r \quad (6.3.24)$$

$$Z_t^w = I_t^w p_t + \tfrac{1}{R_{t+1}} \tfrac{\gamma_w \omega}{\Omega_{t+1}} Z_{t+1}^w + \tfrac{1}{R_{t+1}} (1 - \tfrac{\gamma_w \omega}{\Omega_{t+1}}) \gamma_r Z_{t+1}^r \quad (6.3.25)$$

As shown above, valuations of the worker's non−financial assets are consistent in this life insurance model.

4. Aggregate decision

1) Aggregate consumption and the distribution of wealth

The retirees' total consumption function is derived by combining each retiree's individual consumption functions, and the same procedure is applied to dependents and workers. The mark $\bullet$ is used on the superscripts of all aggregated functions to distinguish them from individual functions. For example, C_t^i indicates the individual consumption function, and $C_t^{i\bullet}$ indicates the aggregate consumption function.

Because individual retirees have the same consumption rate, the aggregate consumption function for retirees can be derived by the sum of individual consumption functions as Eq.(6.3.7) across individual retirees. Let $A_t^{r\bullet}$ be the total financial assets that retirees carry from time t to time t+1. Each retiree earns a return of R_t / γ_r at t, but because the number of surviving retirees is the fraction γ_r of the total, the aggregate gross return on this wealth is R_t. Thus, the total wealth available to retirees at t is $R_t A_t^{r\bullet}$. Let $H_t^{r\bullet}$ be the total human wealth of the current retiree workforce, $S_t^{r\bullet}$ be the sum of the capitalized future social security benefits across retirees at t, and $Z_t^{r\bullet}$ be the sum of the present value of the future insurance expenditures across retirees at t.

Summing Eq.(6.3.7) over individual retirees indicates that the total retirees' consumption at t, $C_t^{r\bullet}$, is given by

$$C_t^{r\bullet} = \varepsilon_t^r \pi_t (R_t A_t^{r\bullet} + H_t^{r\bullet} + S_t^{r\bullet} - Z_t^{r\bullet}), \tag{6.4.1}$$

with

$$H_t^{r\bullet} + S_t^{r\bullet} - Z_t^{r\bullet} = W_t^r L_t^r + E_t^{r\bullet} - I_t^{r\bullet} p_t^r + \tfrac{\gamma_r}{(1+n)R_{t+1}} (H_{t+1}^{r\bullet} + S_{t+1}^{r\bullet} - Z_{t+1}^{r\bullet}), \tag{6.4.2}$$

$$H_t^{r\bullet} = W_t^r L_t^r + \tfrac{\gamma_r}{(1+n)R_{t+1}} H_{t+1}^{r\bullet}, \tag{6.4.3}$$

$$S_t^{r\bullet} = E_t^{r\bullet} + \tfrac{\gamma_r}{(1+n)R_{t+1}} S_{t+1}^{r\bullet}, \tag{6.4.4}$$

$$Z_t^{r\bullet} = I_t^{r\bullet} p_t^r + \tfrac{\gamma_r}{(1+n)R_{t+1}} Z_{t+1}^{r\bullet}, \tag{6.4.5}$$

where

$$I_t^{r\bullet} p_t^r = \tfrac{\nu}{1-\upsilon-\nu} \xi_r W_t (N_t \psi_r - L_t^r). \tag{6.4.6}$$

Equations (6.4.2), (6.4.3), (6.4.4), (6.4.5), (6.4.6) are respectively derived by summing (6.3.9), (6.3.10), (6.3.11), (6.3.12), and (6.3.5) over individual retirees. Because the workforce increases by (1+n) over each period, it is necessary to adjust the value of future non-financial assets at t+1 to the t-point value by using the dividing factor (1+n).

The aggregate dependents' consumption, $C_t^{d\bullet}$ are derived by the same logic used for retirees, as follows.

$$C_t^{d\bullet} = \varepsilon_t^d \pi_t (R_t A_t^{d\bullet} + H_t^{d\bullet} + S_t^{d\bullet} - Z_t^{d\bullet}), \tag{6.4.7}$$

with

$$H_t^{d\bullet} + S_t^{d\bullet} - Z_t^{d\bullet} = W_t^d L_t^d + E_t^{d\bullet} - I_t^{d\bullet} p_t^d + \tfrac{\gamma_d}{(1+n)R_{t+1}} (H_{t+1}^{d\bullet} + S_{t+1}^{d\bullet} - Z_{t+1}^{d\bullet}), \tag{6.4.8}$$

$$H_t^{d\bullet} = W_t^d L_t^d + \tfrac{\gamma_d}{(1+n)R_{t+1}} H_{t+1}^{d\bullet}, \tag{6.4.9}$$

$$S_t^{d\bullet} = E_t^{d\bullet} + \tfrac{\gamma_d}{(1+n)R_{t+1}} S_{t+1}^{d\bullet}, \tag{6.4.10}$$

$$Z_t^{d\bullet} = I_t^{d\bullet} p_t^d + \tfrac{\gamma_d}{(1+n)R_{t+1}} Z_{t+1}^{d\bullet}, \tag{6.4.11}$$

where

$$I_t^{d\bullet} p_t^d = \tfrac{\nu}{1-\upsilon-\nu} \xi_d W_t (N_t \psi_d - L_t^d). \tag{6.4.12}$$

Individual workers, like retirees and dependents, have the same propensity to consume out of wealth, π_t, so it is easy to derive the aggregate consumption by simply summing the individual consumptions. Therefore, summing (6.3.20) over individual workers indicates that total workers' consumption at t, $C_t^{w\bullet}$, is given by

$$C_t^{w\bullet} = \pi_t (R_t A_t^{w\bullet} + H_t^{w\bullet} + S_t^{w\bullet} - Z_t^{w\bullet}), \tag{6.4.13}$$

with

$$\begin{aligned} H_t^{w\bullet} + S_t^{w\bullet} - Z_t^{w\bullet} = {} & (W_t L_t^w - T_t^I - I_t^{w\bullet} p_t) + \tfrac{1}{(1+n)R_{t+1}} \tfrac{\gamma_w \omega}{\Omega_{t+1}} (H_{t+1}^{w\bullet} + S_{t+1}^{w\bullet} - Z_{t+1}^{w\bullet}) \\ & + \tfrac{1}{(1+n)R_{t+1}} (1 - \tfrac{\gamma_w \omega}{\Omega_{t+1}}) \gamma_r (H_{t+1}^{r\bullet} + S_{t+1}^{r\bullet} - Z_{t+1}^{r\bullet}), \end{aligned} \tag{6.4.14}$$

$$H_t^{w\bullet} = (W_t L_t^w - T_t^I) + \tfrac{1}{(1+n)R_{t+1}} \tfrac{\gamma_w \omega}{\Omega_{t+1}} H_{t+1}^{w\bullet} + \tfrac{1}{(1+n)R_{t+1}} (1 - \tfrac{\gamma_w \omega}{\Omega_{t+1}}) \gamma_r H_{t+1}^{r\bullet}, \tag{6.4.15}$$

$$S_t^{w\bullet} = \tfrac{1}{(1+n)R_{t+1}} \tfrac{\gamma_w \omega}{\Omega_{t+1}} S_{t+1}^{w\bullet} + \tfrac{1}{(1+n)R_{t+1}} (1 - \tfrac{\gamma_w \omega}{\Omega_{t+1}}) \gamma_r S_{t+1}^{r\bullet}, \tag{6.4.16}$$

$$Z_t^{w\bullet} = I_t^{w\bullet} p_t + \tfrac{1}{(1+n)R_{t+1}} \tfrac{\gamma_w \omega}{\Omega_{t+1}} Z_{t+1}^{w\bullet} + \tfrac{1}{(1+n)R_{t+1}} (1 - \tfrac{\gamma_w \omega}{\Omega_{t+1}}) \gamma_r Z_{t+1}^{r\bullet}, \tag{6.4.17}$$

where

$$I_t^{w\bullet} p_t = \tfrac{\nu}{1-\upsilon-\nu} W_t (N_t - L_t^w). \tag{6.4.18}$$

Equations (6.4.14), (6.4.15), (6.4.16), (6.4.17), and (6.4.18) are respectively derived from summing (6.3.22), (6.3.23), (6.3.24), (6.3.25), and (6.3.17) over individual workers. In Eq. (6.4.15), $W_t L_t^w$, of all workers at each time point is pre-tax income, so the total income tax, T_t^I, on all workers must be deducted.

Let $A_t^\bullet$ denote aggregate assets and $\lambda_t^r, \lambda_t^d, \lambda_t^w$ denote the shares of assets held by retirees, dependents, and workers, respectively,

i.e.,

$$\lambda_t^r \equiv A_t^{r\bullet} / A_t^{\bullet}, \quad \lambda_t^d \equiv A_t^{d\bullet} / A_t^{\bullet} \text{ and } \lambda_t^w \equiv (1-\lambda_t^r - \lambda_t^d) \equiv A_t^{w\bullet} / A_t^{\bullet}.$$

Then, (6.4.1), (6.4.7), and (6.4.13) change as

$$C_t^{r\bullet} = \varepsilon_t^r \pi_t (R_t \lambda_t^r A_t^{r\bullet} + H_t^{r\bullet} + S_t^{r\bullet} - Z_t^{r\bullet}), \tag{6.4.19}$$

$$C_t^{d\bullet} = \varepsilon_t^d \pi_t (R_t \lambda_t^d A_t^{d\bullet} + H_t^{d\bullet} + S_t^{d\bullet} - Z_t^{d\bullet}), \tag{6.4.20}$$

$$C_t^{w\bullet} = \pi_t [R_t (1-\lambda_t^r - \lambda_t^d) A_t^{\bullet} + H_t^{w\bullet} + S_t^{w\bullet} - Z_t^{w\bullet}]. \tag{6.4.21}$$

Combining above three consumption functions derived from the three types of agents then yields an aggregate consumption function:

$$\begin{aligned} C_t^{\bullet} = \pi_t [&\{1+(\varepsilon_t^r - 1)\lambda_t^r + (\varepsilon_t^d - 1)\lambda_t^d\} R_t A_t^{\bullet} \\ &+(H_t^{w\bullet} + S_t^{w\bullet} - Z_t^{w\bullet}) + \varepsilon_t^r (H_t^{r\bullet} + S_t^{r\bullet} - Z_t^{r\bullet}) + \varepsilon_t^d (H_t^{d\bullet} + S_t^{d\bullet} - Z_t^{d\bullet})]. \end{aligned} \tag{6.4.22}$$

The individual's state changes with the transition probability, and the distribution of wealth and total consumption of each group change over time. First, the total assets owned by workers at the beginning of period t+1 are the same as those transferred by workers at t into t+1 multiplied by the probability that an agent will remain a worker in the next period, $\gamma_w \omega$.

$$\lambda_{t+1}^w A_{t+1}^{\bullet} = (1-\lambda_{t+1}^r - \lambda_{t+1}^d) A_{t+1}^{\bullet} = \gamma_w \omega [R_t \lambda_t^w A_t^{\bullet} + W_t L_t^w - T_t^i - C_t^{w\bullet} - I_t^{w\bullet} p_t], \tag{6.4.23}$$

which can be rewritten as

$$R_t \lambda_t^w A_t^{\bullet} + W_t L_t^w - T_t^i - C_t^{w\bullet} - I_t^{w\bullet} p_t = \tfrac{(1-\lambda_{t+1}^r - \lambda_{t+1}^d)}{\gamma_w \omega} A_{t+1}^{\bullet}. \tag{6.4.24}$$

The last term in brackets on the RHS of (6.4.23) is the workers' total savings at the end of time t and the workers' assets before the distribution of wealth due to the movement of agents. Therefore, (6.4.24) is derived by merely transforming (6.4.23) to obtain the equations for the retirees' and dependents' share of assets.

Second, retirees' total assets from period t to t+1 depend both on the saving of current retirees at t and the assets of workers at t who retire at t+1. That is, the initial gross assets of the retiree at time t+1 are the sum of the assets accumulated by the existing retiree and the assets held by the new retiree.

$$\lambda_{t+1}^{r} A_{t+1}^{\bullet} = R_t \lambda_t^{r} A_t^{\bullet} + W_t^{r} L_t^{r} + E_t^{r\bullet} - C_t^{r\bullet} - I_t^{r\bullet} p_t^{r} + \gamma_w (1-\omega)[R_t \lambda_t^{w} A_t^{\bullet} + W_t L_t^{w} - T_t^{I} - C_t^{w\bullet} - I_t^{w\bullet} p_t] \quad (6.4.25)$$

Equation (6.4.25) can be rewritten as following equation by using (6.4.24).

$$\lambda_{t+1}^{r} A_{t+1}^{\bullet} = R_t \lambda_t^{r} A_t^{\bullet} + W_t^{r} L_t^{r} + E_t^{r\bullet} - C_t^{r\bullet} - I_t^{r\bullet} p_t^{r} + \gamma_w (1-\omega) \tfrac{(1-\lambda_{t+1}^{r} - \lambda_{t+1}^{d})}{\gamma_w \omega} A_{t+1}^{\bullet} . \quad (6.4.26)$$

When we summarize the equation for λ_{t+1}^{r} after putting (6.4.19) into (6.4.26), then the share of wealth held by retirees evolves according to

$$\begin{aligned} \lambda_{t+1}^{r} &= \omega(1-\varepsilon_t^{r}\pi_t) R_t \lambda_t^{r} \tfrac{A_t^{\bullet}}{A_{t+1}^{\bullet}} \\ &\quad + \omega[W_t^{r} L_t^{r} + E_t^{r\bullet} - \varepsilon_t^{r}\pi_t (H_t^{r\bullet} + S_t^{r\bullet} - Z_t^{r\bullet}) - I_t^{r\bullet} p_t^{r}] \tfrac{1}{A_{t+1}^{\bullet}} + (1-\omega)(1-\lambda_{t+1}^{d}). \end{aligned} \quad (6.4.27)$$

Finally, the dependents' initial total wealth at time t+1 is the sum of the savings in the preceding period and the inheritance and life insurance benefits $I_t^{w2\bullet} p_t^{w2}$ arising from the worker's death.

That is, the wealth held by dependents is as follows:

$$\begin{aligned} \lambda_{t+1}^{d} A_{t+1}^{\bullet} &= R_t \lambda_t^{d} A_t^{\bullet} + W_t^{d} L_t^{d} + E_t^{d\bullet} - C_t^{d\bullet} - I_t^{d\bullet} p_t^{d} \\ &\quad + (1-\gamma_w)[R_t \lambda_t^{w} A_t^{\bullet} + W_t L_t^{w} - T_t^{I} - C_t^{w\bullet} - I_t^{w\bullet} p_t] + I_t^{w2\bullet} p_t^{w2}. \end{aligned} \quad (6.4.28)$$

By using (6.4.24), (6.4.28) can be rewritten as

$$\lambda_{t+1}^{d} A_{t+1}^{\bullet} = R_t \lambda_t^{d} A_t^{\bullet} + W_t^{d} L_t^{d} + E_t^{d\bullet} - C_t^{d\bullet} - I_t^{d\bullet} p_t^{d} + (1-\gamma_w) \tfrac{(1-\lambda_{t+1}^{r} - \lambda_{t+1}^{d})}{\gamma_w \omega} A_{t+1}^{\bullet} + I_t^{w2\bullet} p_t^{w2} \quad (6.4.29)$$

When we summarize the equation for λ_{t+1}^{d} after putting (6.4.20) into (6.4.29), then the share of wealth held by dependents evolves according to

$$\lambda_{t+1}^{d} = \frac{\gamma_w \omega (1-\varepsilon_t^d \pi_t) R_t \lambda_t^d \frac{A_t^\bullet}{A_{t+1}^\bullet} + \gamma_w \omega [W_t^d L_t^d + E_t^{d\bullet} - \varepsilon_t^d \pi_t (H_t^{d\bullet} + S_t^{d\bullet} - Z_t^{d\bullet}) - I_t^{d\bullet} p_t^d + I_t^{w2\bullet} p_t^{w2}] \frac{1}{A_{t+1}^\bullet} + (1-\gamma_w)(1-\lambda_{t+1}^r)}{(1-\gamma_w + \gamma_w \omega)} \,. \qquad (6.4.30)$$

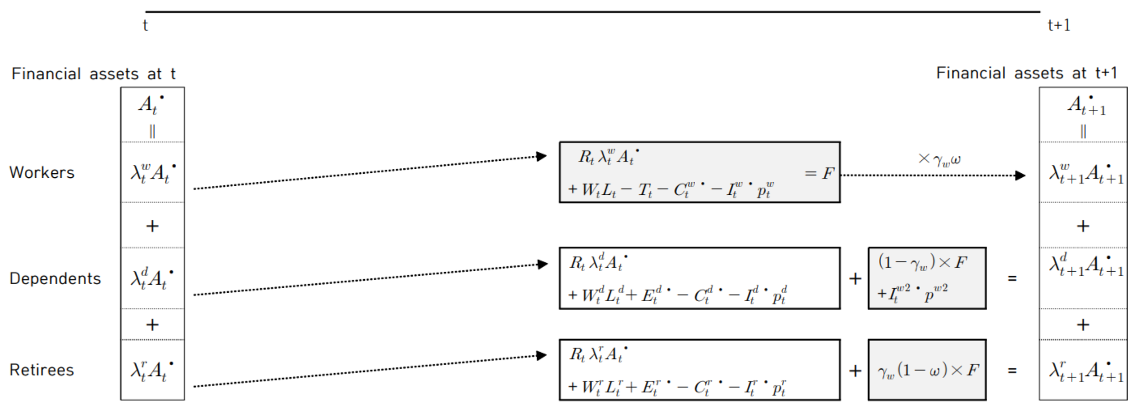

Figure 6−2. Change in the share of assets by economic agents from t to t+1

2) Production and labor supply

As in the previous chapters, Cobb−Douglas production functions are used, and variable labor supply is assumed to mean that individuals can choose to use consumption or leisure given one unit of time.

$$Y_t = (X_t L_t)^{\alpha} K_t^{1-\alpha}, \qquad (6.4.31)$$

$$L_t = L_t^w + \xi_r L_t^r + \xi_d L_t^d \,. \qquad (6.4.32)$$

The aggregate labor supply equations for workers, retirees, and dependents are obtained by simply summing individual labor supply equations (6.3.4), (6.3.16).

$$L_t^w = N_t - \tfrac{(1-\upsilon-\nu)/\upsilon}{W_t} C_t^w \quad (6.4.33)$$

$$L_t^r = N_t \psi_r - \tfrac{(1-\upsilon-\nu)/\upsilon}{\xi_r W_t} C_t^r \quad (6.4.34)$$

$$L_t^d = N_t \psi_d - \tfrac{(1-\upsilon-\nu)/\upsilon}{\xi_d W_t} C_t^d \quad (6.4.35)$$

Equation (6.4.31) implies that the equations for wages and total return on capital are given by

$$W_t = \alpha Y_t / L_t , \quad (6.4.36)$$

$$R_t = (1-\alpha) Y_t / K_t + (1-\delta) . \quad (6.4.37)$$

3) Insurance and governments side

Government implements fiscal and social security policies, and provides social insurance for loss not covered by private insurance, as in the insurance model proposed in Chapter 5. Therefore, each period, the government consumes G_t , provides social insurance G_t^s and pays retirees and dependents a total of social security benefits E_t^r and E_t^d . To finance the expenditure, government issues one period government bonds, B_{t+1} and levies a total of tax, T_t . Tax includes income tax T_t^I levied on workers and consumption tax T_t^C levied on all economic agents.

Thus, the stock of government debt at the beginning of time t+1 is given by

$$B_{t+1} = R_t B_t + G_t + G_t^s + E_t^r + E_t^d - T_t^I - T_t^C . \quad (6.4.38)$$

The ratio of government consumption to output $\bar{g}_t$, the ratio of social security payments to output, $\bar{e}_t^r$, $\bar{e}_t^d$ and the stock of government bonds to output, $\bar{b}_t$ is assumed to be fixed as follows:

$$G_t = \bar{g}_t Y_t, \quad E_t^r = \bar{e}_t^r Y_t, \; E_t^d = \bar{e}_t^d Y_t, \; B_t = \bar{b}_t Y_t. \tag{6.4.39}$$

Social insurance expenditure is defined as

$$G_t^s = \text{Total loss} - -I_t^w p_t - I_t^r p_t^r - I_t^d p_t^d. \tag{6.4.40}$$

The government subsidizes those differences through social insurance when a total loss is not fully covered by private insurance, assuming that the total loss-to-output ratio is constant.

Financial wealth equals the sum of capital and government debt,

$$A_t = K_t + B_t, \tag{6.4.41}$$

and which are the vehicle for saving and the capital intensity evolves as

$$K_{t+1} = Y_t - (C_t - T_t^C) - G_t - G_t^s - I_t^{w1} p_t^{w1} - I_t^r p_t^r - I_t^d p_t^d + (1-\delta)K_t. \tag{6.4.42}$$

To simplify the model, individuals assume that a particular portion of optimal consumption corresponds to consumption tax without considering consumption tax when making decisions to maximize utility. Therefore, in (6.4.42), $(C_t - T_t^C)$ means pure consumption excluding the consumption tax.

4) Steady-state equations

Steady-state equations are derived from the aggregate functions. All quantity variables grow at the growth rate of effective labor force, (1+x)(1+n) and all variables are denoted by the normalized certain variables relative to output in a steady state.

First, let initial values be R, Ω, L_w / N, L_w / L, L_d / L.

Using

$$R = (1-\alpha)k^{-1} + (1-\delta)\,, \tag{6.4.43}$$

produces the initial capital intensity

$$k = (1-\alpha)/(R-1+\delta)\,, \tag{6.4.44}$$

which is tentatively determined. Next, the total tax to output τ can be determined as

$$\tau = [R-(1+x)(1+n)]b + g \ \ + g^S + e^r + e^d\,, \tag{6.4.45}$$

where $\tau = \tau^I + \tau^C$ (τ^I : income tax, τ^C : consumption tax).

The respective steady-state consumption ratio equation for each worker, retiree, and dependent is as follows:

$$\pi = 1 - [(\tfrac{1}{1+x})^{(1-\upsilon-\nu)} R\Omega]^{\sigma-1}\beta^{\sigma}\gamma_w\,, \tag{6.4.46}$$

$$\varepsilon^r \pi = 1 - [(\tfrac{1}{1+x})^{(1-\upsilon-\nu)} R]^{\sigma-1}\beta^{\sigma}\gamma_r\,, \tag{6.4.47}$$

$$\varepsilon^d \pi = 1 - [(\tfrac{1}{1+x})^{(1-\upsilon-\nu)} R]^{\sigma-1}\beta^{\sigma}\gamma_d\,. \tag{6.4.48}$$

In that way, the elasticity of consumption for retirees ε^r and the elasticity of consumption for dependents ε^d are determined.

Workers' non-financial assets can be evaluated as

$$h^w + s^w - z^w = [\alpha \tfrac{L^w}{L} - \tau^I - \iota^w p + \tfrac{(1+x)}{R}\gamma_r(1-\tfrac{\gamma_w\omega}{\Omega})(h^r + s^r - z^r)]/(1-\tfrac{(1+x)}{R}\tfrac{\gamma_w\omega}{\Omega})\,, \tag{6.4.49}$$

$$h^r + s^r - z^r = (\alpha \tfrac{L-L^w-\xi_d L^d}{L} + e^r - \iota^r p^r) / (1 - \tfrac{(1+x)}{R}\gamma_r) , \tag{6.4.50}$$

$$h^d + s^d - z^d = (\alpha \tfrac{\xi_d L^d}{L} + e^d - \iota^d p^d) / (1 - \tfrac{(1+x)}{R}\gamma_d) , \tag{6.4.51}$$

by using the respective recursive non−financial assets equations:

$$\begin{aligned} h^w + s^w - z^w = {} & \alpha \tfrac{L^w}{L} - \tau^I - \iota^w p \\ & + \tfrac{(1+x)}{R}\tfrac{\gamma_w \omega}{\Omega}(h^w + s^w - z^w) + \tfrac{(1+x)}{R}\gamma_r (1 - \tfrac{\gamma_w \omega}{\Omega})(h^r + s^r - z^r) , \end{aligned} \tag{6.4.52}$$

$$h^r + s^r - z^r = \alpha \tfrac{L-L^w-\xi_d L^d}{L} + e^r - \iota^r p^r + \tfrac{(1+x)}{R}\gamma_r (h^r + s^r - z^r) , \tag{6.4.53}$$

$$h^d + s^d - z^d = \alpha \tfrac{\xi_d L^d}{L} + e^d - \iota^d p^d + \tfrac{(1+x)}{R}\gamma_d (h^d + s^d - z^d) . \tag{6.4.54}$$

Next, the asset shares of respective workers, retirees, and dependents can be expressed and solved as

$$\lambda^w = 1 - \lambda^r - \lambda^d , \tag{6.4.55}$$

$$\lambda^r = -\tfrac{(1-\omega)(1+x)(1+n)}{(1+x)(1+n)+\omega R(\varepsilon^r \pi - 1)}\lambda^d + \tfrac{\omega[\alpha \frac{L-L^w-\xi_d L^d}{L} + e^r - \varepsilon^r \pi (h^r + s^r - z^r) - \iota^r p^r](k+b)^{-1} + (1-\omega)(1+x)(1+n)}{(1+x)(1+n)+\omega R(\varepsilon^r \pi - 1)} , \tag{6.4.56}$$

$$\begin{aligned} \lambda^d = {} & -\tfrac{(1-\gamma_w)(1+x)(1+n)}{(1-\gamma_w+\gamma_w \omega)(1+x)(1+n)+\gamma_w \omega R(\varepsilon^d \pi - 1)}\lambda^r \\ & + \tfrac{\gamma_w \omega[\alpha \frac{\xi_d L^d}{L} + e^d - \varepsilon^d \pi (h^d + s^d - z^d) - \iota^d p^d + \iota^{w2} p^{w2}](k+b)^{-1} + (1-\gamma_w)(1+x)(1+n)}{(1-\gamma_w+\gamma_w \omega)(1+x)(1+n)+\gamma_w \omega R(\varepsilon^d \pi - 1)} . \end{aligned} \tag{6.4.57}$$

The steady−state version of each aggregate consumption amount can be written as

$$c^w = \pi[(1 - \lambda^r - \lambda^d) R(k+b) + h^w + s^w - z^w] , \tag{6.4.58}$$

$$c^r = \varepsilon^r \pi[\lambda^r R(k+b) + h^r + s^r - z^r] , \tag{6.4.59}$$

$$c^d = \varepsilon^d \pi[\lambda^d R(k+b) + h^d + s^d - z^d] , \tag{6.4.60}$$

and then the aggregation equation will be

$$
\begin{aligned}
c = \pi[\{1+(\varepsilon^r-1)\lambda^r+(\varepsilon^d-1)\lambda^d\}R(k+b) \\
+(h^w+s^w-z^w)+\varepsilon^r(h^r+s^r-z^r)+\varepsilon^d(h^d+s^d-z^d)],
\end{aligned}
\tag{6.4.61}
$$

which can be used in calculating intensity capital such that

$$
\left[(1+x)(1+n)-1+\delta\right]k = 1-(c-\tau^C)-g-g^s-\iota^{w1}p^{w1}-\iota^r p^r-\iota^d p^d . \tag{6.4.62}
$$

Placing (6.4.61) into (6.4.62), then we have

$$
\begin{aligned}
&\left[(1+x)(1+n)-1+\delta\right]k = 1-\pi[\{1+(\varepsilon^r-1)\lambda^r+(\varepsilon^d-1)\lambda^d\}R(k+b) \\
&+(h^w+s^w-z^w)+\varepsilon^r(h^r+s^r-z^r)+\varepsilon^d(h^d+s^d-z^d)]+\tau^C-g-g^s-\iota^{w1}p^{w1}-\iota^r p^r-\iota^d p^d
\end{aligned}
\tag{6.4.63}
$$

That produces new recursive values

$$
R = \frac{1-[(1+x)(1+n)-1+\delta]k-\iota^{w1}p^{w1}-\iota^r p^r-\iota^d p^d-[\pi\{(h^w+s^w-z^w)+\varepsilon^r(h^r+s^r-z^r)+\varepsilon^d(h^d+s^d-z^d)\}-\tau^C]-g-g^s}{\pi[1+(\varepsilon^r-1)\lambda^r+(\varepsilon^d-1)\lambda^d](k+b)} , \tag{6.4.64}
$$

and the worker's risk adjustment factor and labor supply of agents are as follows:

$$
\Omega = \gamma_w[\omega+(1-\omega)\tfrac{1}{\gamma_r}(\varepsilon^r)^{\frac{1}{1-\sigma}}\chi_r] , \tag{6.4.65}
$$

$$
\tfrac{L^w}{L} = \tfrac{N}{L} - \tfrac{(1-\upsilon-\nu)}{\alpha\upsilon}c^w , \tag{6.4.66}
$$

$$
\tfrac{L^r}{L} = \tfrac{N\psi_r}{L} - \tfrac{(1-\upsilon-\nu)}{\xi_r\alpha\upsilon}c^r , \tag{6.4.67}
$$

$$
\tfrac{L^d}{L} = \tfrac{N\psi_d}{L} - \tfrac{(1-\upsilon-\nu)}{\xi_d\alpha\upsilon}c^d , \tag{6.4.68}
$$

$$
\tfrac{L}{N} = (1+\xi_r\psi_r+\xi_d\psi_d)[1+\tfrac{(1-\upsilon-\nu)}{\alpha\upsilon}c]^{-1} , \tag{6.4.69}
$$

and can be used to minimize the distance measure of the initial and new values of R, Ω, L_w/N, L_w/L, L_d/L.

5. Sub−conclusion

This chapter describes the life insurance model, an extension of the insurance model in Chapter 5. The life insurance model is theoretically sophisticated than insurance model in that it reflects a more realistic population structure, asset allocation, diversification of insurance type. It is also possible to analyze the impacts of life and non−life insurance on the economy in a steady state.

The life insurance model gives dependents besides workers and retirees the role of economic agents. As the possibility of workers' death is imposed, workers' conditions are diversified, and risk adjustment factors and valuation factors used in decision making are changed. It allows a dependent on receiving inheritance and death benefits from life insurance in the event of a worker's death. Therefore, it is possible to analyze the impact of life insurance purchases on the transfer of assets between workers and dependents.

Chapter 7. Applications of Insurance Model

In this chapter, Section 1 derives the initial steady−state values by calibration. Section 2 examines the effects of increasing the proportion of life insurance on economic agents' behavior and the economy. Furthermore, Section 3 summarizes the implications of this work.

1. Initial steady−state values

Most parameters retain the values presented in Gertler's (1999) paper and set additional parameter values associated with the new concepts introduced in this model: the worker's survival rate γ_w, the dependent's survival rate γ_d, the preference parameter for insurance ν, and the productivity of a unit of labor supplied by a dependent relative to a worker ξ_d. the worker's risk probability in non−life insurance p^{w1}, the worker's risk probability in life insurance p^{w2}, the retiree's risk probability in non−life insurance p^{r}.

Assuming a 45−year−old representative worker who retires at the age of 65, the remaining working periods are 20 years($\frac{1}{1-\gamma_w\omega}$ =20), the probability of remaining in the labor force conditional on survival is 0.95. Similarly, assuming a representative 72.5−year−old retiree who dies at the age of 85, the retiree's remaining life expectancy is 12.5 years($\frac{1}{1-\gamma_r}$ =12.5), with a survival rate of 0.92. For a representative dependent with a life expectancy of 20 years($\frac{1}{1-\gamma_d}$ =20), the survival rate is 0.95. Meanwhile, Gertler sets the consumption preference parameter to 0.4 and the leisure preference parameter to 0.6 in his paper following Cooley's business cycle literature (1995).

Table 7-1. Description and value of parameters

Parameter	Description	Value
n	Workforce growth rate	0.01
$\gamma_w\omega$	Probability of remaining in the labor force conditional on survival	0.95
γ_r	Probability of surviving for a retiree	0.92
γ_d	Probability of surviving for a dependent	0.95
υ	Preference parameter for consumption	0.40
ν	Preference parameter for insurance	0.04
$1-\upsilon-\nu$	Preference parameter for leisure	0.56
β	Subjective discount rate	1
ρ	Curvature parameter	−3
σ	Elasticity of intertemporal substitution	0.25
ξ_r	Relative productivity of retirees to workers	0.6
ξ_d	Relative productivity of dependents to workers	0.7
α	Labor income share	0.667
δ	Capital depreciation rate	0.1
x	Productivity growth rate	0.01
b	Government debt to GDP	0.25
g	Government consumption to GDP	0.20
e^r	Social security payments to GDP for retirees	0.03
e^d	Social security payments to GDP for dependents	0.001
p^{w1}	Risk probability of a worker in nonlife insurance	0.01
p^{w2}	Risk probability of a worker in life insurance	0.003
p^r	Risk probability of a retiree in nonlife insurance	0.03

Table 7−2 reports the initial steady−state values when the parameters in Table 7−1 are used as inputs.

Table 7−2. Initial steady−state values of basic exogenous variables

Variable	Description	Value	Value per capita
k	Capital stock	2.053	
K / XL	Capital stock per unit of effective labor	2.941	
R	Gross return on capital	1.062	
π	Propensity to consume for a worker	0.081	
$\varepsilon^r \pi$	Propensity to consume for a retiree	0.117	
$\varepsilon^d \pi$	Propensity to consume for a dependent	0.088	
ε^r	Retiree's consumption elasticity	1.442	
ε^d	Dependent's consumption elasticity	1.087	
Ω	Risk adjustment factor	1.056	
h^w	Human wealth	3.154	4.959
h^r		0.597	1.796
h^d		0.099	3.107
s^w	Social security wealth	0.140	0.220
s^r		0.231	0.696
s^d		0.011	0.338
z^w	Present value of future insurance expenditure	0.335	0.527
z^r		0.150	0.450
z^d		0.021	0.652
$h^w + s^w - z^w$	Non−financial assets	2.959	4.652
$h^r + s^r - z^r$		0.678	2.041
$h^d + s^d - z^d$		0.089	3.107
$\lambda^w = 1 - \lambda^r - \lambda^d$	Share of assets	0.566	0.890
λ^r		0.377	1.136
λ^d		0.057	1.779
$\tau = \tau^I + \tau^C$	Total tax	0.241	

τ^I	Total income tax	0.179	
τ^C	Total consumption tax	0.061	
c^w	Consumption	0.353	0.555
c^r		0.188	0.565
c^d		0.020	0.631
$c = c^w + c^r + c^d$		0.561	
$\iota^{w1} p^{w1}$	Insurance expenditure	0.034	0.053
$\iota^{w2} p^{w2}$		0.002	0.003
$\iota^r p^r$		0.019	0.056
$\iota^d p^d$		0.002	0.063
$\tau^I / \alpha \frac{L^w}{L}$	Workers' tax burden	0.308	
L^w / N	Labor supply as a fraction of total time endowment	0.541	
$L^r / N\psi_r$		0.222	
$L^d / N\psi_d$		0.254	

Notes: 1) In the superscript or subscript of variables, w denotes workers, r denotes retirees, and d denotes dependents. 2) Quantitative variables such as capital stock, non−financial assets, tax, consumption, insurance expenditure are represented as the normalized variables relative to output.

In steady state, the capital−output ratio is 2.053, the capital stock per unit of effective labor is 2.941, and the total return on capital is 1.062. The consumption rate of retirees and dependents tend to more than workers, and the consumption elasticity of both retirees and dependents is more than one.

Of total financial assets, the workers' share is about 56.6%; the retirees' share is about 37.7%; the dependents' share is about 5.7%. The proportion of total tax revenue to GDP is 24.0%, of which the income tax is 17.9%, and the consumption tax is 6.1%, which is about 10% of the total consumption amounts. For workers, the tax burden to labor income is about 30.8%; the ratio of consumption to post−tax income is 87.6%; the ratio of consumption, including insurance expenditure to post−tax income, is 96.3%. The model shows that only workers pay income taxes, so the proportion of workers' consumption to their after−tax income is relatively high, so they have less room for capital accumulation than other groups.

Also, workers' labor supply is about 54.1%, and the labor supply of retirees is only about 20% because they have lower unit labor productivity and lower wages than workers. It reflects the fact that retirees have accumulated more economic wealth than average workers.

The fourth column of Table 7−2 shows the steady−state values per capita for easy comparison among groups. The population proportion for each group is obtained using the number of retirees/dependents to the number of workers. The population proportion of workers is 63.6%, that of retirees is 33.2%, and that of dependents is 3.2%.

Concerning the human wealth per capita, the value is 4.959 for a worker, 1.796 for a retiree, and 3.107 for a dependent, respectively. Workers have the most extended working period and the highest efficiency per unit, so the value of human assets per person is also the highest. Retired workers have shorter working periods than workers, and only 60% of their per capita labor efficiency, making them the lowest value of human assets per person. Dependents have a per−unit labor efficiency of 70 percent compared with that of a worker, though they have a more extended working period than retirees.

For the social security wealth per capita, the value is 0.22 for a worker, 0.696

for a retiree, and 0.338 for a dependent. Workers' social security wealth is the sum for all workers alive at t of the capitalized future social security benefits they can expect during retirement. Similarly, retirees' social security wealth is the total capitalized value of social security benefits of all retirees alive at t. Thus, the capitalization period for workers is more than twice that of retirees. Considering the discounted period and the population's size, it is plausible that the social security wealth per retiree is more considerable than the social security wealth per worker.

Finally, the share of financial assets per capita is 0.89 for a worker, 1.136 for a retiree, and 1.779 for a dependent. To efficiently analyze the results, we recalculated an adjusted value of the share of financial assets per capita by setting a retiree's value to one. The adjusted values for a worker, a retiree, and a dependent are 0.784, 1.000, and 1.566, respectively. The financial assets of retirees and dependents are existing assets and assets transferred from workers in the event of retirement or death. The share of assets held by dependents accounts for only about 5 percent of the total assets, but due to inheritance and the death benefits from life insurance upon a worker's death, the share of financial assets per dependent is higher than that of a worker or a retiree.

2. Effects of increasing the proportion of life insurance

In the basic scenario, the proportion of non-life insurance purchased by workers was set at 95%, and that of life insurance was set at 5 %. This section analyzes how increasing the proportion of life insurance purchased by workers affects the steady-state capital and gross interest rates and changes economic agents' life-cycle behavior.

With workers' total insurance expenditures fixed, the relative proportion of life insurance changes from 0% to 10%. As a result, capital stock increases by 3.11% from 2.89 to 2.98, and gross return decreases 0.003%p from 1.064 to 1.061. Non-life insurance is used for loss compensation purposes, but life insurance affects both asset transfer and savings. In other words, the total life insurance expenditure paid by a worker is transferred to the death benefits that dependents receive upon the worker's death. Therefore, total life insurance expenditure becomes an asset of the dependents upon the worker's death, and the dependents spend some of it and save the rest.

The critical aspect of this analysis is the change in the assets of dependents, who are the direct beneficiaries of any increase in the portion that workers spend on life insurance, as shown in Table 7-4 and Figure 7-2. The dependents' share of assets increases by 2.5%p from 4.4 percent to 6.9 percent. The share of assets per dependent increases significantly, from 1.198 when the proportion of life insurance is zero to 1.936 when the proportion of life insurance is 10 percent. Dependents rely heavily on inheritance and death benefits because they have lower wages than workers and lower social security benefits than retirees. Dependents who have little or no property to inherit from their workers have no choice but to live in poverty. Accordingly, life insurance is necessary to eliminate these risks in the event of an employee's death and to ensure a minimum level of assets for the dependents.

Table 7−3. Effects of the proportion of life insurance on capital and gross return

Proportion of life insurance	Capital stock per unit of effective labor (K/XL)	Gross return on capital (R)
0.00	2.891	1.0643
0.02	2.911	1.0635
0.04	2,931	1.0628
0.06	2.949	1.0621
0.08	2.965	1.0615
0.10	2.981	1.0609
	△0.090	▽0.003

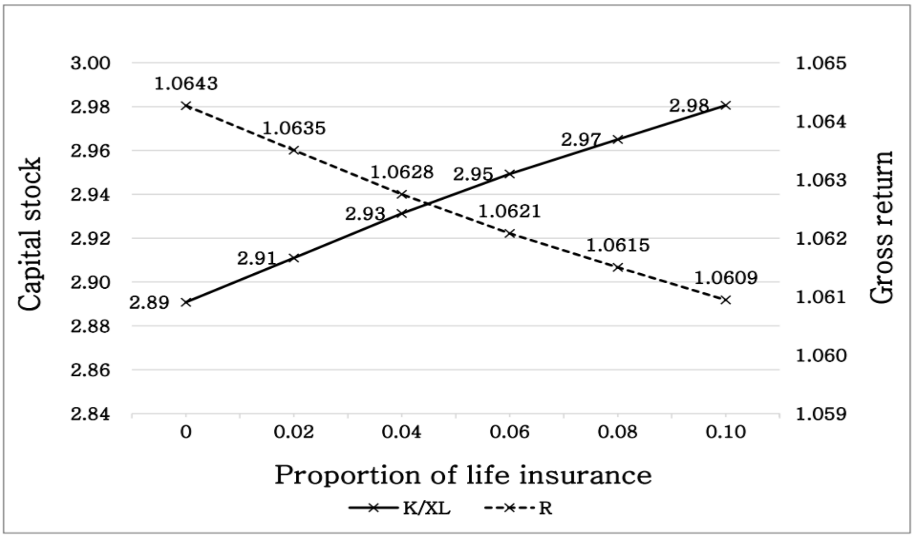

Figure 7−1. Effects of the proportion of life insurance on capital and gross return

Table 7-4. Effects of the proportion of life insurance on the share of assets

	Share of assets (λ^i)			Adjusted share of assets per capita (λ^i / q_i)		
Proportion of life insurance	Workers (w)	Retirees (r)	Dependents (d)	Workers (w)	Retirees (r)	Dependents (d)
0.00	0.572	0.384	0.044	0.779	1.000	1.198
0.02	0.570	0.381	0.049	0.781	1.000	1.345
0.04	0.567	0.379	0.054	0.783	1.000	1.493
0.06	0.565	0.376	0.059	0.785	1.000	1.640
0.08	0.562	0.374	0.064	0.786	1.000	1.788
0.10	0.560	0.371	0.069	0.788	1.000	1.936
	▽1.2%p	▽1.3%p	△2.5%p			

Notes: i = w (workers), r (retirees), d (dependents).

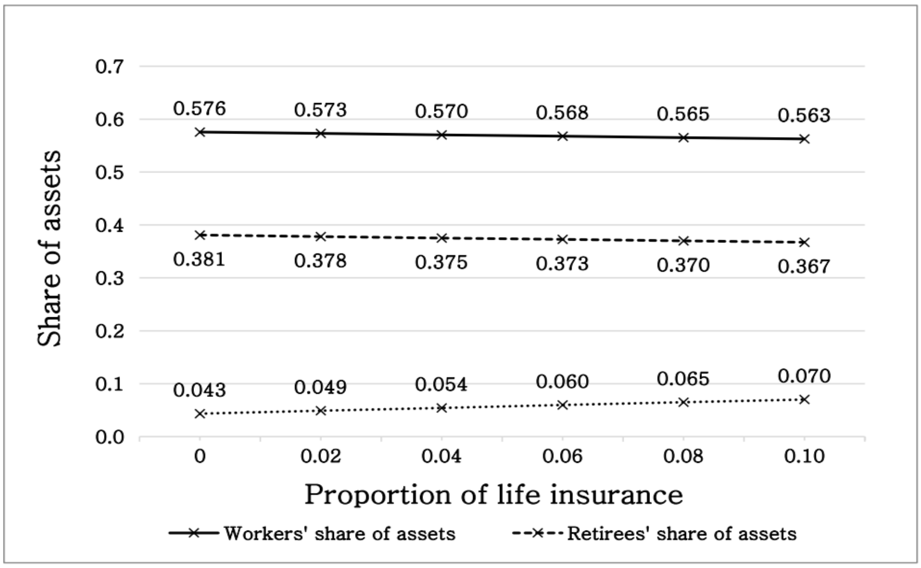

Figure 7-2. Effects of the proportion of life insurance on share of assets

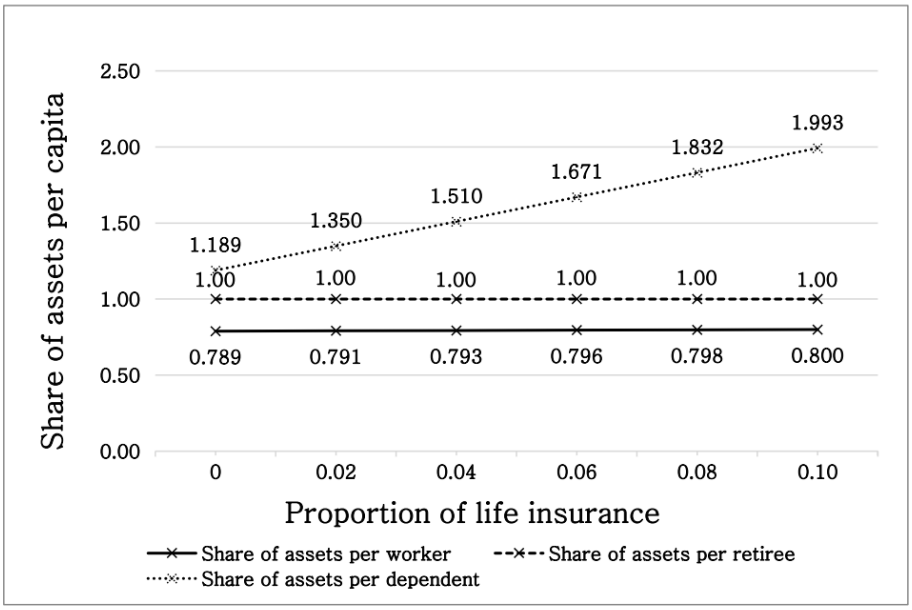

Figure 7-3. Effects of the proportion of life insurance on the share of assets per capita

To summarize the sensitivity analyses above, the increase in the proportion of life insurance products purchased increases the dependents' asset share and reduces labor supply. When the total insurance purchase expenditure is constant, a higher percentage of workers' life insurance purchases leads to more assets transferred to dependents, which leads to an increase in capital stock and a decrease in total return on capital.

Chapter 8. Conclusions

1. Summary

Unlike traditional OLG models, this study incorporates insurance into the OLG model and analyzes insurance effects on economic agents' life−cycle behavior and the economy. This paper studies based on Gertler's (1999) model and mainly focuses on three issues: rederivation, modification, and development of insurance model within Gertler's OLG framework. The contributions and results of this study are as follows.

First, the rederivation of Gertler's model has made economic agents' decision problems more evident, including variable labor supply. The results of the application of the rederived model to the Korean economy are as follows. Reinforcement of social security causes intergenerational transfer of assets, resulting in labor supply and consumption changes for workers and retirees. In particular, workers are greatly affected by a tax burden that changes with social security. The increase in social security benefits has a relatively large negative wealth effect on retirees' labor supply. In addition, it has resulted in lower capital intensity and higher gross return on capital. As the retirement period is prolonged, the population structure changes to the increase in retirees' proportion. Since individuals reduce their consumption rate in preparation for their retirement, the total consumption to GDP ratio decreases, and capital stock increases. An extension of the retirement age increases retirees' proportion relatively less and mitigates changes in labor supply and share of financial assets for both economic agents. On the contrary, the consumption−to−GDP ratio decrements are larger than before, leading to a more considerable rise in the capital stock than when there is only an aging effect.

Second, the modification of Gertler's model results in a more accurate formula for risk adjustment factor and more clearly defined its role. The adjustment factor plays a role in adjusting the discount rate in Gertler's model, and in the proposed model, it plays a role in adjusting the transition probability. It also makes valuations in workers' non−financial assets consistent. In Gertler's model, the valuation of social security wealth contains an ambiguous transition factor, and it overestimates the steady−state value of social security wealth, compared with the result of the proposed model. Consistency in the valuation of non−financial assets obtained through the modified model also applies usefully in subsequent insurance models. That is, consistency in valuations is ensured regardless of the number of assets, making it easier to apply to the extended model. In addition, it is found that the steady state value in Gertler's model is slightly underestimated for the risk adjustment factor and overestimated for the non−financial assets−to−output ratio more than in the proposed model.

Third, an insurance model based on modified Gertler's OLG model is proposed. In this model, individuals purchase non−life insurance products as the optimal choice to maximize their utility, and the government provides public insurance for losses that are not covered by partial private insurance, which is called social insurance. Because the government requires workers to pay taxes to provide indemnification for agents, the proportion of private or social insurance coverage to total loss affects workers' tax burden. If the ratio of social insurance coverage to total loss from 0 to 1, the percentage of total tax to GDP increases by 4.8%p, and workers' tax burden to labor income increases by 7%p. The advantage of the insurance model is that it recognizes losses from particular financial events that the existing economic models do not recognize.

Fourth, a life insurance model is presented. In order to introduce life insurance, basic assumptions about population composition and transition probabilities are modified so that dependents can receive inheritance and death insurance payments when workers die. The life insurance model is theoretically more sophisticated than the insurance model in that it reflects a more realistic population structure and asset allocation, and diversified insurance types. It is also possible to analyze the microeconomic and macroeconomic effects of insurance in steady-state, unlike traditional dynamic equilibrium models. As the proportion of life insurance purchases increases, the proportion of dependents' assets increases, and labor supply decreases. Also, if a worker dies, the total life insurance expenditure becomes the dependent's asset, and the remaining assets of the dependent after consumption contribute to the accumulation of capital.

2. Further studies

Gertler's OLG model is tractable and useful for economic analysis, but additional problems still need to be addressed. First, it is necessary to develop a generalized OLG model assuming heterogeneous agents with idiosyncratic characteristics for work probability, mortality rate, wage, etc. In Gertler's model, each generation has the advantage of being simple to analyze because homogeneous agents represent it, but there is a limit to the practical analysis of social security and fiscal policy in the real economy. Therefore, further research is needed to build a heterogeneous agent OLG model, such as deriving methods for estimating transition probabilities using the realistic average length of remaining work and retirement life of heterogeneous agents.

Second, cross-country analysis should be done using the calibration method developed to practically implement the economic analysis model. In several countries, social security and fiscal policy have a significant impact on cross-country macroeconomic variables, such as interest rates, capital intensity, and labor supply, depending on the parameters reasonably estimated by the proposed calibration method.

Third, the relationship between private and social insurance needs to be reflected in the model more clearly and elaborately. In this study, there is a limit that financial losses that are not fully covered by partial private insurance are not reflected in utility. The model addresses such financial losses by introducing social insurance, and the demand for private insurance reduces the demand for social insurance, so private and social insurance are substitute relations on the assumption that a constant ratio of total losses to GDP. However, it is controversial whether the relationship between private and social insurance is substitutes or complements because it can vary depending on the country's situation, such as the insurance system, policy, economy, culture, and many other factors. Therefore, it is necessary to develop a more sophisticated model by establishing and reflecting the relationship between private and social insurance.

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Appendix

<Appendix A> Elastic labor supply

1) Retiree−decision problem

Maximize

$$V_t^r = \left\{[(C_t^r)^{\upsilon}(1-l_t^r)^{1-\upsilon}]^{\rho} + \beta\gamma(V_{t+1}^r)^{\rho}\right\}^{1/\rho}, \tag{A.1.1}$$

subject to

$$A_{t+1}^r = \tfrac{R_t}{\gamma} A_t^r + W_t^r l_t^r + E_t - C_t^r. \tag{A.1.2}$$

Using Lagrange multiplier μ, the optimization problem can be rewritten as

$$L = V_t^r - \mu(A_{t+1}^r - \tfrac{R_t}{\gamma} A_t^r - W_t^r l_t^r - E_t + C_t^r). \tag{A.1.3}$$

Let us partially differentiate L with respect to C_t^r, l_t^r and A_{t+1}^r. From the three partial derivatives, i.e.

$$\frac{\partial L}{\partial C_t^r} = (V_t^r)^{1-\rho}\upsilon(C_t^r)^{\upsilon\rho-1}(1-l_t^r)^{(1-\upsilon)\rho} - \mu = 0, \tag{A.1.4}$$

$$\frac{\partial L}{\partial l_t^r} = -(V_t^r)^{1-\rho}(1-\upsilon)(C_t^r)^{\upsilon\rho}(1-l_t^r)^{(1-\upsilon)\rho-1} + \mu W_t^r = 0, \tag{A.1.5}$$

$$\frac{\partial L}{\partial A_{t+1}^r} = (V_t^r)^{1-\rho}\beta\gamma\tfrac{\partial V_{t+1}^r}{\partial A_{t+1}^r}(V_{t+1}^r)^{\rho-1} - \mu = 0, \tag{A.1.6}$$

we have

$$1 - l_t^r = \tfrac{1-\upsilon}{\upsilon} C_t^r / W_t^r. \tag{A.1.7}$$

From (A.1.4) and (A.1.6)

$$\upsilon(C_t^r)^{\upsilon\rho-1}(1-l_t^r)^{(1-\upsilon)\rho} = \beta\gamma \tfrac{\partial V_{t+1}^r}{\partial A_{t+1}^r}(V_{t+1}^r)^{\rho-1}, \tag{A.1.8}$$

$$\mu = \upsilon(C_t^r)^{\upsilon\rho-1}(1-l_t^r)^{(1-\upsilon)\rho}(V_t^r)^{1-\rho}. \tag{A.1.9}$$

Applying the Envelope Theorem with parameter A_t^r,

$$\frac{dV_t^r}{dA_t^r} = \frac{\partial L}{\partial A_t^r} = \mu \tfrac{R_t}{\gamma} = \tfrac{R_t}{\gamma}\upsilon(C_t^r)^{\upsilon\rho-1}(1-l_t^r)^{(1-\upsilon)\rho}(V_t^r)^{1-\rho}. \tag{A.1.10}$$

From (A.10), we have

$$\frac{dV_{t+1}^r}{dA_{t+1}^r} = \tfrac{R_{t+1}}{\gamma}\upsilon(C_{t+1}^r)^{\upsilon\rho-1}(1-l_{t+1}^r)^{(1-\upsilon)\rho}(V_{t+1}^r)^{1-\rho}. \tag{A.1.11}$$

Let us guess the form of V_t^r

$$V_t^r = (\varepsilon_t\pi_t)^{-\frac{1}{\rho}}(C_t^r)^{\upsilon}(1-l_t^r)^{1-\upsilon}, \tag{A.1.12}$$

and plug (A.1.7) in (A.1.12)

$$V_t^r = (\varepsilon_t\pi_t)^{-\frac{1}{\rho}} C_t^r (\tfrac{1-\upsilon}{\upsilon W_t^r})^{1-\upsilon}. \tag{A.1.13}$$

From (A.1.13), we have

$$V_{t+1}^r = (\varepsilon_{t+1}\pi_{t+1})^{-\frac{1}{\rho}} C_{t+1}^r (\tfrac{1-\upsilon}{\upsilon W_{t+1}^r})^{1-\upsilon}. \tag{A.1.14}$$

Placing (A.1.11), (A.1.14) into (A.1.8)

$$\upsilon(C_t^r)^{\rho-1}(\tfrac{1-\upsilon}{\upsilon W_t^r})^{(1-\upsilon)\rho} = \beta R_{t+1}\upsilon(\tfrac{1-\upsilon}{\upsilon W_{t+1}^r})^{1-\upsilon}[C_{t+1}^r(\tfrac{1-\upsilon}{\upsilon W_{t+1}^r})^{1-\upsilon}]^{\rho-1}, \tag{A.1.15}$$

which is rewritten as

$$(C_{t+1}^r)^{1-\rho} = (\tfrac{W_t^r}{W_{t+1}^r})^{(1-\upsilon)\rho} R_{t+1}\beta(C_t^r)^{1-\rho}, \tag{A.1.16}$$

or which is also expresses as

$$C_{t+1}^r = [(\tfrac{W_t^r}{W_{t+1}^r})^{(1-\upsilon)\rho} R_{t+1}\beta]^{\sigma} C_t^r, \tag{A.1.17}$$

where $\sigma = 1/(1-\rho)$ and $W_t^r = \xi W_t$.

Next, let us guess a solution of the form:

$$C_t^r = \varepsilon_t \pi_t (\tfrac{R_t}{\gamma} A_t^r + H_t^r + S_t^r). \tag{A.1.18}$$

Check that

$$C_{t+1}^r = \varepsilon_{t+1} \pi_{t+1} (\tfrac{R_{t+1}}{\gamma} A_{t+1}^r + H_{t+1}^r + S_{t+1}^r). \tag{A.1.19}$$

Placing (A.1.18), (A.1.19) into (A.1.17), we obtain

$$\varepsilon_{t+1}\pi_{t+1}(\tfrac{R_{t+1}}{\gamma} A_{t+1}^r + H_{t+1}^r + S_{t+1}^r) = [(\tfrac{W_t^r}{W_{t+1}^r})^{(1-\upsilon)\rho} R_{t+1}\beta]^{\sigma} \varepsilon_t \pi_t (\tfrac{R_t}{\gamma} A_t^r + H_t^r + S_t^r). \tag{A.1.20}$$

Now, substituting (A.1.13), (A.1.14) into value function (A.1.1), and replacing C_{t+1}^r to the form of C_t^r using the relation (A.1.17), then we obtain

$$[(\varepsilon_t \pi_t)^{-\frac{1}{\rho}} C_t^r (\tfrac{1-\upsilon}{\upsilon W_t^r})^{1-\upsilon}]^{\rho} = [C_t^r (\tfrac{1-\upsilon}{\upsilon W_t^r})^{1-\upsilon}]^{\rho} + \beta\gamma[(\varepsilon_{t+1}\pi_{t+1})^{-\frac{1}{\rho}}((\tfrac{W_t^r}{W_{t+1}^r})^{(1-\upsilon)\rho} R_{t+1}\beta)^{\sigma} C_t^r (\tfrac{1-\upsilon}{\upsilon W_{t+1}^r})^{1-\upsilon}]^{\rho}, \tag{A.1.21}$$

which can be written as

$$(\varepsilon_t \pi_t)^{-1} = 1 + [(\tfrac{W_t^r}{W_{t+1}^r})^{1-\upsilon} R_{t+1}]^{\sigma-1} \beta^{\sigma} \gamma (\varepsilon_{t+1}\pi_{t+1})^{-1}. \tag{A.1.22}$$

Eq. (A.1.22) is also expressed as

$$\varepsilon_t \pi_t = 1 - [(\tfrac{W_t^r}{W_{t+1}^r})^{1-\upsilon} R_{t+1}]^{\sigma-1} \beta^{\sigma} \gamma \tfrac{\varepsilon_t \pi_t}{\varepsilon_{t+1}\pi_{t+1}}. \tag{A.1.23}$$

From (A.1.23), we have

$$\varepsilon_{t+1}\pi_{t+1} = [(\tfrac{W_t^r}{W_{t+1}^r})^{1-\upsilon} R_{t+1}]^{\sigma-1} \beta^{\sigma} \gamma \varepsilon_t \pi_t / (1 - \varepsilon_t \pi_t). \tag{A.1.24}$$

To confirm a solution for the value function, conjecture that

$$V_t^r = \Delta_t^r (C_t^r)^{\upsilon} (1 - l_t^r)^{1-\upsilon} = \Delta_t^r (C_t^r)^{\upsilon} (\tfrac{1-\upsilon}{\upsilon W_t^r})^{1-\upsilon}. \tag{A.1.25}$$

Then, to obtain an expression for Δ_t^r, substitute the conjectured solution for

V_t^r into the objective to obtain

$$\Delta_t^r C_t^r (\tfrac{1-\upsilon}{\upsilon W_t^r})^{1-\upsilon} = \left[\left\{ C_t^r (\tfrac{1-\upsilon}{\upsilon W_t^r})^{1-\upsilon} \right\}^\rho + \beta\gamma \left\{ \Delta_{t+1}^r C_{t+1}^r (\tfrac{1-\upsilon}{\upsilon W_{t+1}^r})^{1-\upsilon} \right\}^\rho \right]^{1/\rho}. \quad \text{(A.1.26)}$$

Here, placing (A.1.17) into (A.1.26), (A.1.26) can be rewritten as

$$\left\{ \Delta_t^r C_t^r (\tfrac{1-\upsilon}{\upsilon W_t^r})^{1-\upsilon} \right\}^\rho = \left\{ C_t^r (\tfrac{1-\upsilon}{\upsilon W_t^r})^{1-\upsilon} \right\}^\rho + \beta\gamma \left[\Delta_{t+1}^r \left\{ (\tfrac{W_t^r}{W_{t+1}^r})^{(1-\upsilon)\rho} R_{t+1} \beta \right\}^\sigma C_t^r (\tfrac{1-\upsilon}{\upsilon W_{t+1}^r})^{1-\upsilon} \right]^\rho. \quad \text{(A.1.27)}$$

Thus, applying that $\sigma\rho + 1 = \sigma$ and $\sigma\rho = \sigma - 1$, we have

$$(\Delta_t^r)^\rho = 1 + [(\tfrac{W_t^r}{W_{t+1}^r})^{(1-\upsilon)} R_{t+1}]^{\sigma-1} \beta^\sigma \gamma (\Delta_{t+1}^r)^\rho. \quad \text{(A.1.28)}$$

Now, checking that (A.1.28) is identical to (A.1.22), we obtain

$$\Delta_t^r = (\varepsilon_t \pi_t)^{-\frac{1}{\rho}}. \quad \text{(A.1.29)}$$

Thus,

$$V_t^r = (\varepsilon_t \pi_t)^{-\frac{1}{\rho}} C_t^r (\tfrac{1-\upsilon}{\upsilon W_t^r})^{(1-\upsilon)}. \quad \text{(A.1.30)}$$

2) Worker-decision problem

Maximize

$$V_t^w = \left\{ [(C_t^w)^\upsilon (1-l_t^w)^{1-\upsilon}]^\rho + \beta[\omega V_{t+1}^w + (1-\omega) V_{t+1}^r]^\rho \right\}^{1/\rho}, \quad \text{(A.2.1)}$$

subject to

$$A_{t+1}^w = R_t A_t^w + W_t\, l_t^w - C_t^w, \quad \text{(A.2.2)}$$

where W_t is after-tax wage.

Using Lagrange multiplier μ, the optimization problem can be written as

$$L = V_t^w - \mu(C_t^w + A_{t+1}^w - R_t A_t^w - W_t\, l_t^w)\,. \tag{A.2.3}$$

Let us partially differentiate L with respect to C_t^w, l_t^w and A_{t+1}^w. From the three partial derivatives, i.e.

$$\frac{\partial L}{\partial C_t^w} = (V_t^w)^{1-\rho}\upsilon(C_t^w)^{\upsilon\rho-1}(1-l_t^w)^{(1-\upsilon)\rho} - \mu = 0\,, \tag{A.2.4}$$

$$\frac{\partial L}{\partial l_t^w} = -(V_t^w)^{1-\rho}(1-\upsilon)(C_t^w)^{\upsilon\rho}(1-l_t^w)^{(1-\upsilon)\rho-1} + \mu W_t = 0\,, \tag{A.2.5}$$

$$\frac{\partial L}{\partial A_{t+1}^w} = (V_t^w)^{1-\rho}[\omega V_{t+1}^w + (1-\omega)V_{t+1}^r]^{\rho-1}\beta(\omega \tfrac{\partial V_{t+1}^w}{\partial A_{t+1}^w} + (1-\omega)\tfrac{\partial V_{t+1}^r}{\partial A_{t+1}^r}) - \mu = 0\,, \tag{A.2.6}$$

we have

$$1 - l_t^w = \tfrac{1-\upsilon}{\upsilon} C_t^w / W_t\,. \tag{A.2.7}$$

From (A.2.4) and (A.2.6)

$$\upsilon(C_t^w)^{\upsilon\rho-1}(1-l_t^w)^{(1-\upsilon)\rho} = [\omega V_{t+1}^w + (1-\omega)V_{t+1}^r]^{\rho-1}\beta(\omega \tfrac{\partial V_{t+1}^w}{\partial A_{t+1}^w} + (1-\omega)\tfrac{\partial V_{t+1}^r}{\partial A_{t+1}^r}) \tag{A.2.8}$$

$$\mu = \upsilon(C_t^w)^{\upsilon\rho-1}(1-l_t^w)^{(1-\upsilon)\rho}(V_t^w)^{1-\rho}\,. \tag{A.2.9}$$

Applying the Envelope Theorem with parameter A_t^w,

$$\frac{dV_t^w}{dA_t^w} = \frac{\partial L}{\partial A_t^w} = \mu R_t = \upsilon(C_t^w)^{\upsilon\rho-1}(1-l_t^w)^{(1-\upsilon)\rho}(V_t^w)^{1-\rho}R_t\,. \tag{A.2.10}$$

From (A.2.10), we have

$$\frac{\partial V_{t+1}^w}{\partial A_{t+1}^w} = \upsilon(C_{t+1}^w)^{\upsilon\rho-1}(1-l_{t+1}^w)^{(1-\upsilon)\rho}(V_{t+1}^w)^{1-\rho}R_{t+1}\,. \tag{A.2.11}$$

Let us guess the form of V_t^w is analogous to the form of V_t^r

$$V_t^w = (\pi_t)^{-\frac{1}{\rho}}(C_t^w)^{\upsilon}(1-l_t^w)^{1-\upsilon} = (\pi_t)^{-\frac{1}{\rho}} C_t^w (\tfrac{1-\upsilon}{\upsilon W_t})^{1-\upsilon}\,. \tag{A.2.12}$$

From (A.2.12), we have

$$V_{t+1}^{w}=(\pi_{t+1})^{-\frac{1}{\rho}}C_{t+1}^{w}(\tfrac{1-\upsilon}{\upsilon W_{t+1}})^{1-\upsilon}, \tag{A.2.13}$$

$$V_{t+1}^{r}=(\varepsilon_{t+1}\pi_{t+1})^{-\frac{1}{\rho}}C_{t+1}^{r}(\tfrac{1-\upsilon}{\upsilon W_{t+1}^{r}})^{1-\upsilon}. \tag{A.1.14}$$

Note that

$$\frac{dV_{t+1}^{j}}{dA_{t+1}^{j}}=\upsilon(C_{t+1}^{j})^{\upsilon\rho-1}(1-l_{t+1}^{j})^{(1-\upsilon)\rho}(V_{t+1}^{j})^{1-\rho}R_{t+1}. \qquad \text{for j = w, r} \tag{A.2.14}$$

Placing (A.1.14), (A.2.13) and (A.2.14) into (A.2.8), then,

$$\begin{aligned}\upsilon(C_{t}^{w})^{\rho-1}(\tfrac{1}{W_{t}})^{(1-\upsilon)\rho}&=[(\tfrac{1-\upsilon}{W_{t+1}})^{1-\upsilon}]^{\rho-1}[\omega(\pi_{t+1})^{-\frac{1}{\rho}}C_{t+1}^{w}+(1-\omega)(\varepsilon_{t+1}\pi_{t+1})^{-\frac{1}{\rho}}C_{t+1}^{r}(\tfrac{W_{t+1}}{W_{t+1}^{r}})^{1-\upsilon}]^{\rho-1}\\&\times\beta R_{t+1}\upsilon(\tfrac{1-\upsilon}{W_{t+1}})^{1-\upsilon}[\omega(\pi_{t+1})^{-\frac{1-\rho}{\rho}}+(1-\omega)(\tfrac{W_{t+1}}{W_{t+1}^{r}})^{1-\upsilon}(\varepsilon_{t+1}\pi_{t+1})^{-\frac{1-\rho}{\rho}}]\end{aligned} \tag{A.2.15}$$

Simplify the (A.2.15)

$$(C_{t}^{w})^{\rho-1}=(\tfrac{W_{t}}{W_{t+1}})^{(1-\upsilon)\rho}[\omega C_{t+1}^{w}+(1-\omega)(\varepsilon_{t+1})^{-\frac{1}{\rho}}C_{t+1}^{r}(\tfrac{W_{t+1}}{W_{t+1}^{r}})^{1-\upsilon}]^{\rho-1}\beta R_{t+1}[\omega+(1-\omega)(\tfrac{W_{t+1}}{W_{t+1}^{r}})^{1-\upsilon}(\varepsilon_{t+1})^{-\frac{1-\rho}{\rho}}]. \tag{A.2.16}$$

Let us define that $\xi=\frac{W_{t+1}^{r}}{W_{t+1}}$, $\chi=(\frac{1}{\xi})^{1-\upsilon}$ and $\sigma=1/(1-\rho)$

$$\Omega_{t+1}=\omega+(1-\omega)(\varepsilon_{t+1})^{\frac{1}{1-\sigma}}\chi. \tag{A.2.17}$$

Then, (A.2.17) is rewritten as

$$\omega C_{t+1}^{w}+(1-\omega)\chi(\varepsilon_{t+1})^{\frac{\sigma}{1-\sigma}}C_{t+1}^{r}=[(\tfrac{W_{t}}{W_{t+1}})^{1-\upsilon}]^{\sigma-1}(R_{t+1}\Omega_{t+1}\beta)^{\sigma}C_{t}^{w}. \tag{A.2.18}$$

Now, let us guess a consumption solution of the form:

$$C_{t}^{w}=\pi_{t}(R_{t}A_{t}^{w}+H_{t}^{w}+S_{t}^{w}), \tag{A.2.19}$$

$$C_{t+1}^{w}=\pi_{t+1}(R_{t+1}A_{t+1}^{w}+H_{t+1}^{w}+S_{t+1}^{w}). \tag{A.2.20}$$

Applying (A.2.19), (A.2.20), (A.1.19) to (A.2.18), then we have

$$\pi_{t+1}[\omega(R_{t+1}A_{t+1}^{w}+H_{t+1}^{w}+S_{t+1}^{w})+(1-\omega)(\varepsilon_{t+1})^{\frac{1}{1-\sigma}}\chi(\tfrac{R_{t+1}}{\gamma}A_{t+1}^{r}+H_{t+1}^{r}+S_{t+1}^{r})]$$
$$=[(\tfrac{W_t}{W_{t+1}})^{1-\upsilon}]^{\sigma-1}(R_{t+1}\Omega_{t+1}\beta)^{\sigma}\pi_t(R_tA_t^{w}+H_t^{w}+S_t^{w}). \quad (A.2.21)$$

Now, placing (A.2.12), (A.2.13), (A.1.14) into value function (A.2.1), we obtain

$$[(\pi_t)^{-\frac{1}{\rho}}C_t^{w}(\tfrac{1-\upsilon}{\upsilon W_t})^{1-\upsilon}]^{\rho}=[C_t^{w}(\tfrac{1-\upsilon}{\upsilon W_t})^{1-\upsilon}]^{\rho}+\beta[\omega(\pi_{t+1})^{-\frac{1}{\rho}}C_{t+1}^{w}(\tfrac{1-\upsilon}{\upsilon W_{t+1}})^{1-\upsilon}+(1-\omega)(\varepsilon_{t+1}\pi_{t+1})^{-\frac{1}{\rho}}C_{t+1}^{r}(\tfrac{1-\upsilon}{\upsilon W_{t+1}^{r}})^{1-\upsilon}]^{\rho},$$

(A.2.22)

which can be written as

$$(\pi_t)^{-1}=1+\beta\left\{(\tfrac{W_t}{W_{t+1}})^{1-\upsilon}[\omega C_{t+1}^{w}+(1-\omega)\chi(\varepsilon_{t+1})^{\frac{\sigma}{1-\sigma}}C_{t+1}^{r}]/C_t^{w}\right\}^{\frac{\sigma-1}{\sigma}}(\pi_{t+1})^{-1} \quad (A.2.23)$$

and placing (A.2.18) into (A.2.23). Hence, we have

$$\pi_t=1-[(\tfrac{W_t}{W_{t+1}})^{1-\upsilon}R_{t+1}\Omega_{t+1}]^{\sigma-1}\beta^{\sigma}(\pi_t/\pi_{t+1})\,. \quad (A.2.24)$$

From (A.2.24), we have

$$\pi_{t+1}=[(\tfrac{W_t}{W_{t+1}})^{1-\upsilon}R_{t+1}\Omega_{t+1}]^{\sigma-1}\beta^{\sigma}[\pi_t/(1-\pi_t)]\,. \quad (A.2.25)$$

To confirm a solution for the value function, conjecture that

$$V_t^{w}=\Delta_t^{w}(C_t^{w})^{\upsilon}(1-l_t^{w})^{1-\upsilon}=\Delta_t^{w}(C_t^{w})^{\upsilon}(\tfrac{1-\upsilon}{\upsilon W_t})^{1-\upsilon}\,. \quad (A.2.26)$$

Then, to obtain an expression for Δ_t^{w}, substitute the conjectured solution for V_t^{w} into the objective to obtain

$$\Delta_t^{w}C_t^{w}(\tfrac{1-\upsilon}{\upsilon W_t})^{1-\upsilon}=\left[(C_t^{w}(\tfrac{1-\upsilon}{\upsilon W_t})^{1-\upsilon})^{\rho}+\beta\left\{\omega\Delta_{t+1}^{w}C_{t+1}^{w}(\tfrac{1-\upsilon}{\upsilon W_{t+1}})^{1-\upsilon}+(1-\omega)\Delta_{t+1}^{r}C_{t+1}^{r}(\tfrac{1-\upsilon}{\upsilon W_{t+1}^{r}})^{1-\upsilon}\right\}^{\rho}\right]^{1/\rho}. \quad (A.2.27)$$

Here, placing (A.2.18) into (A.2.27), (A.2.27) can be rewritten as

$$\left(\Delta_t^{w}C_t^{w}(\tfrac{1-\upsilon}{\upsilon W_t})^{(1-\upsilon)}\right)^{\rho}=\left(C_t^{w}(\tfrac{1-\upsilon}{\upsilon W_t})^{(1-\upsilon)}\right)^{\rho}+\beta\left\{\Delta_{t+1}^{w}\left[\left(\frac{W_t}{W_{t+1}}\right)^{(1-\upsilon)\rho}R_{t+1}\Omega_{t+1}\beta\right]^{\sigma}C_t^{w}(\tfrac{1-\upsilon}{\upsilon W_{t+1}})^{(1-\upsilon)}\right\}^{\rho}. \quad (A.2.28)$$

Thus, applying that $\sigma\rho+1=\sigma$ and $\sigma\rho=\sigma-1$, we have

$$\left(\Delta_t^w\right)^{\rho}=1+[\left(\tfrac{W_t}{W_{t+1}}\right)^{(1-\upsilon)}R_{t+1}\Omega_{t+1}]^{\sigma-1}\beta^{\sigma}\left(\Delta_{t+1}^w\right)^{\rho}. \tag{A.2.29}$$

Now, checking that (A.2.29) is identical to (A.2.23), we obtain

$$\Delta_t^w=\left(\pi_t\right)^{-\frac{1}{\rho}}. \tag{A.2.30}$$

Thus,

$$V_t^w=\left(\pi_t\right)^{-\frac{1}{\rho}}C_t^w(\tfrac{1-\upsilon}{\upsilon W_t})^{(1-\upsilon)}. \tag{A.2.31}$$

<Appendix B> Insurance model

1) Retiree-decision problem

Maximize

$$V_t^r = \left\{[(C_t^r)^{\upsilon}(I_t^r)^{\nu}(1-l_t^r)^{1-\upsilon-\nu}]^{\rho} + \beta\gamma(V_{t+1}^r)^{\rho}\right\}^{1/\rho}, \tag{B.1.1}$$

subject to

$$A_{t+1}^r = \tfrac{R_t}{\gamma}A_t^r + W_t^r l_t^r + E_t - C_t^r - I_t^r p_t^r\,. \tag{B.1.2}$$

Using Lagrange multiplier μ, the optimization problem can be rewritten as

$$L = V_t^r - \mu(A_{t+1}^r - \tfrac{R_t}{\gamma}A_t^r - W_t^r l_t^r - E_t + C_t^r + I_t^r p_t^r)\,. \tag{B.1.3}$$

Let us partially differentiate L with respect to C_t^r, I_t^r, l_t^r and A_{t+1}^r. From the four partial derivatives, i.e.

$$\frac{\partial L}{\partial C_t^r} = (V_t^r)^{1-\rho}\upsilon(C_t^r)^{\upsilon\rho-1}(I_t^r)^{\nu\rho}(1-l_t^r)^{(1-\upsilon-\nu)\rho} - \mu = 0\,, \tag{B.1.4}$$

$$\frac{\partial L}{\partial I_t^r} = (V_t^r)^{1-\rho}\nu(C_t^r)^{\upsilon\rho-1}(I_t^r)^{\nu\rho-1}(1-l_t^r)^{(1-\upsilon-\nu)\rho} - \mu p_t^r = 0\,, \tag{B.1.5}$$

$$\frac{\partial L}{\partial l_t^r} = -(V_t^r)^{1-\rho}(1-\upsilon-\nu)(C_t^r)^{\upsilon\rho}(I_t^r)^{\nu\rho}(1-l_t^r)^{(1-\upsilon-\nu)\rho-1} + \mu W_t^r = 0\,, \tag{B.1.6}$$

$$\frac{\partial L}{\partial A_{t+1}^r} = (V_t^r)^{1-\rho}\beta\gamma\tfrac{\partial V_{t+1}^r}{\partial A_{t+1}^r}(V_{t+1}^r)^{\rho-1} - \mu = 0\,, \tag{B.1.7}$$

we have

$$I_t^r p_t^r = \tfrac{\nu}{\upsilon}C_t^r\,, \tag{B.1.8}$$

$$1 - l_t^r = \tfrac{1-\upsilon-\nu}{\upsilon}C_t^r / W_t^r\,. \tag{B.1.9}$$

From (B.1.4) and (B.1.7)

$$\upsilon (C_t^r)^{\upsilon\rho-1}(I_t^r)^{\nu\rho}(1-l_t^r)^{(1-\upsilon-\nu)\rho} = \beta\gamma \tfrac{\partial V_{t+1}^r}{\partial A_{t+1}^r}(V_{t+1}^r)^{\rho-1}, \tag{B.1.10}$$

$$\mu = \upsilon (C_t^r)^{\upsilon\rho-1}(I_t^r)^{\nu\rho}(1-l_t^r)^{(1-\upsilon-\nu)\rho}(V_t^r)^{1-\rho}. \tag{B.1.11}$$

Applying the Envelope Theorem with parameter A_t^r,

$$\frac{dV_t^r}{dA_t^r} = \frac{\partial L}{\partial A_t^r} = \mu \tfrac{R_t}{\gamma} = \tfrac{R_t}{\gamma}\upsilon (C_t^r)^{\upsilon\rho-1}(I_t^r)^{\nu\rho}(1-l_t^r)^{(1-\upsilon-\nu)\rho}(V_t^r)^{1-\rho}. \tag{B.1.12}$$

From (B.1.12), we have

$$\frac{\partial V_{t+1}^r}{\partial A_{t+1}^r} = \tfrac{R_{t+1}}{\gamma}\upsilon (C_{t+1}^r)^{\upsilon\rho-1}(I_{t+1}^r)^{\nu\rho}(1-l_{t+1}^r)^{(1-\upsilon-\nu)\rho}(V_{t+1}^r)^{1-\rho}. \tag{B.1.13}$$

Let us guess the form of V_t^r

$$V_t^r = (\varepsilon_t \pi_t)^{-\frac{1}{\rho}}(C_t^r)^{\upsilon}(I_t^r)^{\nu}(1-l_t^r)^{1-\upsilon-\nu}, \tag{B.1.14}$$

and plug (B.1.8), (B.1.9) in (B.1.14), then

$$V_t^r = (\varepsilon_t \pi_t)^{-\frac{1}{\rho}} C_t^r (\tfrac{\nu}{\upsilon p_t^r})^{\nu}(\tfrac{1-\upsilon-\nu}{\upsilon W_t^r})^{(1-\upsilon-\nu)}. \tag{B.1.15}$$

From (B.1.15), we have

$$V_{t+1}^r = (\varepsilon_{t+1} \pi_{t+1})^{-\frac{1}{\rho}} C_{t+1}^r (\tfrac{\nu}{\upsilon p_{t+1}^r})^{\nu}(\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^r})^{(1-\upsilon-\nu)}. \tag{B.1.16}$$

Placing (B.1.8), (B.1.9), and (B.1.16) into (B.1.13)

$$\frac{\partial V_{t+1}^r}{\partial A_{t+1}^r} = \tfrac{R_{t+1}}{\gamma}\upsilon (\tfrac{\nu}{\upsilon p_{t+1}})^{\nu}(\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^r})^{(1-\upsilon-\nu)}(\varepsilon_{t+1}\pi_{t+1})^{-\frac{1-\rho}{\rho}}. \tag{B.1.17}$$

Substituting (B.1.16), (B.1.17) into (B.1.10)

$$\upsilon (C_t^r)^{\upsilon\rho-1}(I_t^r)^{\nu\rho}(1-l_t^r)^{(1-\upsilon-\nu)\rho} = \beta R_{t+1}\upsilon (\tfrac{\nu}{\upsilon p_{t+1}^r})^{\nu}(\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^r})^{1-\upsilon-\nu}[C_{t+1}^r(\tfrac{\nu}{\upsilon p_{t+1}^r})^{\nu}(\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^r})^{1-\upsilon-\nu}]^{\rho-1}, \tag{B.1.18}$$

which is rewritten as

$$C_{t+1}^r = [(\tfrac{W_t^r}{W_{t+1}^r})^{(1-\upsilon-\nu)\rho}(\tfrac{p_t^r}{p_{t+1}^r})^{\nu\rho} R_{t+1}\beta]^{\sigma} C_t^r , \quad \text{(B.1.19)}$$

where $\sigma = 1/(1-\rho)$ and $W_t^r = \xi W_t$.

Next, let us guess a solution of the form:

$$C_t^r = \varepsilon_t \pi_t (\tfrac{R_t}{\gamma} A_t^r + H_t^r + S_t^r - Z_t^r) . \quad \text{(B.1.20)}$$

Check that

$$C_{t+1}^r = \varepsilon_{t+1}\pi_{t+1}(\tfrac{R_{t+1}}{\gamma} A_{t+1}^r + H_{t+1}^r + S_{t+1}^r - Z_{t+1}^r) . \quad \text{(B.1.21)}$$

Placing (B.1.20), (B.1.21) into (B.1.19), we can obtain

$$\varepsilon_{t+1}\pi_{t+1}[\tfrac{R_{t+1}}{\gamma} A_{t+1}^r + (H_{t+1}^r + S_{t+1}^r - Z_{t+1}^r)] = [(\tfrac{W_t^r}{W_{t+1}^r})^{(1-\upsilon-\nu)\rho}(\tfrac{p_t^r}{p_{t+1}^r})^{\nu\rho} R_{t+1}\beta]^{\sigma} \varepsilon_t \pi_t [\tfrac{R_t}{\gamma} A_t^r + (H_t^r + S_t^r - Z_t^r)] ,$$

(B.1.22)

Now, placing (B.1.15), (B.1.16) into value function (B.1.1), then

$$[(\varepsilon_t \pi_t)^{-\frac{1}{\rho}} C_t^r (\tfrac{\nu}{\upsilon p_t^r})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^r})^{1-\upsilon-\nu}]^{\rho} = [C_t^r (\tfrac{\nu}{\upsilon p_t^r})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^r})^{1-\upsilon-\nu}]^{\rho} + \beta\gamma[(\varepsilon_{t+1}\pi_{t+1})^{-\frac{1}{\rho}} C_{t+1}^r (\tfrac{\nu}{\upsilon p_{t+1}^r})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^r})^{1-\upsilon-\nu}]^{\rho} .$$

(B.1.23)

Replacing C_{t+1}^r to the form of C_t^r using the relation (B.1.19)

$$\begin{aligned}
&[(\varepsilon_t \pi_t)^{-\frac{1}{\rho}} C_t^r (\tfrac{\nu}{\upsilon p_t^r})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^r})^{1-\upsilon-\nu}]^{\rho} \\
&= [C_t^r (\tfrac{\nu}{\upsilon p_t^r})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^r})^{1-\upsilon-\nu}]^{\rho} + \beta\gamma[(\varepsilon_{t+1}\pi_{t+1})^{-\frac{1}{\rho}} ((\tfrac{W_t^r}{W_{t+1}^r})^{(1-\upsilon-\nu)\rho}(\tfrac{p_t^r}{p_{t+1}^r})^{\nu\rho} R_{t+1}\beta)^{\sigma} C_t^r (\tfrac{\nu}{\upsilon p_{t+1}^r})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^r})^{1-\upsilon-\nu}]^{\rho} ,
\end{aligned}$$

(B.1.24)

which can be written as

$$(\varepsilon_t \pi_t)^{-1} = 1 + [(\tfrac{W_t^r}{W_{t+1}^r})^{1-\upsilon-\nu}(\tfrac{p_t^r}{p_{t+1}^r})^{\nu} R_{t+1}]^{\sigma-1} \beta^{\sigma} \gamma (\varepsilon_{t+1}\pi_{t+1})^{-1} , \quad \text{(B.1.25)}$$

$$\varepsilon_t \pi_t = 1 - [(\tfrac{W_t^r}{W_{t+1}^r})^{1-\upsilon-\nu}(\tfrac{p_t^r}{p_{t+1}^r})^{\nu} R_{t+1}]^{\sigma-1} \beta^{\sigma} \gamma \tfrac{\varepsilon_t \pi_t}{\varepsilon_{t+1}\pi_{t+1}} , \quad \text{(B.1.26)}$$

From (B.1.26), we have

$$\varepsilon_t \pi_t = [(\tfrac{W_t^r}{W_{t+1}^r})^{1-\upsilon-\nu}(\tfrac{p_t^r}{p_{t+1}^r})^{\nu} R_{t+1}]^{\sigma-1}\beta^{\sigma}\gamma \tfrac{\varepsilon_t \pi_t}{(1-\varepsilon_t \pi_t)}, \tag{B.1.27}$$

and multiply both sides by the same equations as $R_{t+1}(1-\varepsilon_t \pi_t)(\frac{R_t}{\gamma}A_t^r + H_t^r + S_t^r - Z_t^r)$,

and then we have

$$\begin{aligned}&\varepsilon_{t+1}\pi_{t+1}[R_{t+1}(1-\varepsilon_t \pi_t)(\tfrac{R_t}{\gamma}A_t^r + H_t^r + S_t^r - Z_t^r)]\\ &=[(\tfrac{W_t^r}{W_{t+1}^r})^{1-\upsilon-\nu}(\tfrac{p_t^r}{p_{t+1}^r})^{\nu}]^{\sigma-1}(R_{t+1}\beta)^{\sigma}\gamma\varepsilon_t \pi_t(\tfrac{R_t}{\gamma}A_t^r + H_t^r + S_t^r - Z_t^r).\end{aligned} \tag{B.1.28}$$

Here, note that the RHS of (B.1.28) is equal to the product of γ and the RHS of (B.1.22). Hence,

$$R_{t+1}(1-\varepsilon_t \pi_t)(\tfrac{R_t}{\gamma}A_t^r + H_t^r + S_t^r - Z_t^r) = \gamma(\tfrac{R_{t+1}}{\gamma}A_{t+1}^r + H_{t+1}^r + S_{t+1}^r - Z_{t+1}^r), \tag{B.1.29}$$

i.e.

$$(1-\varepsilon_t \pi_t)(\tfrac{R_t}{\gamma}A_t^r + H_t^r + S_t^r - Z_t^r) = A_{t+1}^r + \tfrac{\gamma}{R_{t+1}}(H_{t+1}^r + S_{t+1}^r - Z_{t+1}^r). \tag{B.1.30}$$

It is also possible to express the LHS of (B.1.30) as

$$\begin{aligned}&(1-\varepsilon_t \pi_t)(\tfrac{R_t}{\gamma}A_t^r + H_t^r + S_t^r - Z_t^r)\\ &=(\tfrac{R_t}{\gamma}A_t^r + W_t^r l_t^r + E_t - C_t^r - I_t^r p_t^r) + (H_t^r - W_t^r l_t^r) + (S_t^r - E_t) - (Z_t^r - I_t^r p_t^r).\end{aligned} \tag{B.1.31}$$

Then, from (B.1.30) and (B.1.31), following equation holds

$$\begin{aligned}&A_{t+1}^r + \tfrac{\gamma}{R_{t+1}}(H_{t+1}^r + S_{t+1}^r - Z_{t+1}^r)\\ &=(\tfrac{R_t}{\gamma}A_t^r + W_t^r l_t^r + E_t - C_t^r - I_t^r p_t^r) + (H_t^r - W_t^r l_t^r) + (S_t^r - E_t) - (Z_t^r - I_t^r p_t^r).\end{aligned} \tag{B.1.32}$$

It is obvious that from (B.1.2)

$$\tfrac{R_t}{\gamma}A_t^r + W_t^r l_t^r + E_t - C_t^r - I_t^r p_t^r = A_{t+1}^r. \tag{B.1.33}$$

Thus, we obtain

$$\tfrac{\gamma}{R_{t+1}}(H_{t+1}^r + S_{t+1}^r - Z_{t+1}^r) = (H_t^r - W_t^r l_t^r) + (S_t^r - E_t) - (Z_t^r - I_t^r p_t^r), \tag{B.1.34}$$

i.e.

$$H_t^r + S_t^r - Z_t^r = W_t^r l_t^r + E_t - I_t^r p_t^r + \tfrac{\gamma}{R_{t+1}}(H_{t+1}^r + S_{t+1}^r - Z_{t+1}^r)\,. \qquad \text{(B.1.35)}$$

It is not necessary that the following equations

$$H_t^r = W_t^r l_t^r + \tfrac{\gamma}{R_{t+1}} H_{t+1}^r\,, \qquad \text{(B.1.36)}$$

$$S_t^r = E_t + \tfrac{\gamma}{R_{t+1}} S_{t+1}^r\,, \qquad \text{(B.1.37)}$$

$$Z_t^r = I_t^r p_t^r + \tfrac{\gamma}{R_{t+1}} Z_{t+1}^r \qquad \text{(B.1.38)}$$

hold true. Eq. (B.1.35) is applied to derive aggregate demand, supply and steady state endogenous variables. But, it is convenient to use equations (B.1.36), (B.1.37) and (B.1.38) for calculation.

To confirm a solution for the value function, conjecture that

$$V_t^r = \Delta_t^r (C_t^r)^{\upsilon} (I_t^r)^{\nu} (1 - l_t^r)^{1-\upsilon-\nu} = \Delta_t^r (C_t^r)^{\upsilon} (\tfrac{\nu}{\upsilon p_t^r})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^r})^{(1-\upsilon-\nu)}\,. \qquad \text{(B.1.39)}$$

Then, to obtain an expression for Δ_t^r, substitute the conjectured solution for V_t^r into the objective to obtain

$$\Delta_t^r C_t^r (\tfrac{\nu}{\upsilon p_t^r})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^r})^{(1-\upsilon-\nu)} = \left[\left\{ C_t^r (\tfrac{\nu}{\upsilon p_t^r})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^r})^{(1-\upsilon-\nu)} \right\}^{\rho} + \beta\gamma \left\{ \Delta_{t+1}^r C_{t+1}^r (\tfrac{\nu}{\upsilon p_{t+1}^r})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^r})^{(1-\upsilon-\nu)} \right\}^{\rho} \right]^{1/\rho}. \qquad \text{(B.1.40)}$$

Here, placing (B.1.19) into (B.1.40), (B.1.40) can be rewritten as

$$\begin{aligned} &\left\{ \Delta_t^r C_t^r (\tfrac{\nu}{\upsilon p_t^r})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^r})^{(1-\upsilon-\nu)} \right\}^{\rho} \\ &= \left\{ C_t^r (\tfrac{\nu}{\upsilon p_t^r})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^r})^{(1-\upsilon-\nu)} \right\}^{\rho} + \beta\gamma \left[\Delta_{t+1}^r \left\{ (\tfrac{W_t^r}{W_{t+1}^r})^{(1-\upsilon-\nu)\rho} (\tfrac{p_t^r}{p_{t+1}^r})^{\nu\rho} R_{t+1}\beta \right\}^{\sigma} C_t^r (\tfrac{\nu}{\upsilon p_t^r})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^r})^{(1-\upsilon-\nu)} \right]^{\rho}. \end{aligned} \qquad \text{(B.1.41)}$$

Thus, applying that $\sigma\rho + 1 = \sigma$ and $\sigma\rho = \sigma - 1$, we have

$$(\Delta_t^r)^\rho = 1 + [(\tfrac{W_t^r}{W_{t+1}^r})^{(1-\upsilon-\nu)}(\tfrac{p_t^r}{p_{t+1}^r})^\nu R_{t+1}]^{\sigma-1}\beta^\sigma\gamma(\Delta_{t+1}^r)^\rho . \qquad \text{(B.1.42)}$$

Now, checking that (B.1.42) is identical to (B.1.25), we obtain

$$\Delta_t^r = (\varepsilon_t \pi_t)^{-\frac{1}{\rho}} . \qquad \text{(B.1.43)}$$

Thus,

$$V_t^r = \left(\varepsilon_t \pi_t\right)^{-\frac{1}{\rho}} C_t^r (\tfrac{\nu}{\upsilon p_t^r})^\nu (\tfrac{1-\upsilon-\nu}{\upsilon W_t^r})^{(1-\upsilon-\nu)} . \qquad \text{(B.1.44)}$$

2) Worker−decision problems

Maximize

$$V_t^w = \left\{[(C_t^w)^\upsilon (I_t^w)^\nu (1-l_t^w)^{1-\upsilon-\nu}]^\rho + \beta[\omega V_{t+1}^w + (1-\omega)V_{t+1}^r]^\rho\right\}^{1/\rho} , \qquad \text{(B.2.1)}$$

subject to

$$A_{t+1}^w = R_t A_t^w + W_t\, l_t^w - C_t^w - I_t^w p_t \, , \qquad \text{(B.2.2)}$$

where p_t is insurance premium for basic benefit and W_t is after−tax wage.

Using Lagrange multiplier μ, the optimization problem can be written as

$$L = V_t^w - \mu(A_{t+1}^w - R_t A_t^w - W_t\, l_t^w + C_t^w + I_t^w p_t) \qquad \text{(B.2.3)}$$

Let us partially differentiate L with respect to C_t^w, I_t^w, l_t^w and A_{t+1}^w. From the four partial derivatives, i.e.

$$\frac{\partial L}{\partial C_t^w} = (V_t^w)^{1-\rho}\upsilon(C_t^w)^{\upsilon\rho-1}(I_t^w)^{\nu\rho}(1-l_t^w)^{(1-\upsilon-\nu)\rho} - \mu = 0 , \qquad \text{(B.2.4)}$$

$$\frac{\partial L}{\partial I_t^w} = (V_t^w)^{1-\rho}\nu(C_t^w)^{\upsilon\rho-1}(I_t^w)^{\nu\rho-1}(1-l_t^w)^{(1-\upsilon-\nu)\rho} - \mu p_t = 0 , \qquad \text{(B.2.5)}$$

$$\frac{\partial L}{\partial l_t^w} = -(V_t^w)^{1-\rho}(1-\upsilon-\nu)(C_t^w)^{\upsilon\rho}(I_t^w)^{\nu\rho}(1-l_t^w)^{(1-\upsilon-\nu)\rho-1} + \mu W_t = 0, \tag{B.2.6}$$

$$\frac{\partial L}{\partial A_{t+1}} = (V_t^w)^{1-\rho}[\omega V_{t+1}^w + (1-\omega)V_{t+1}^r]^{\rho-1}\beta[\omega \tfrac{\partial V_{t+1}^w}{\partial A_{t+1}} + (1-\omega)\tfrac{\partial V_{t+1}^r}{\partial A_{t+1}}] - \mu = 0, \tag{B.2.7}$$

we have

$$I_t^w p_t = \tfrac{\nu}{\upsilon} C_t^w, \tag{B.2.8}$$

$$1 - l_t^w = \tfrac{1-\upsilon-\nu}{\upsilon} C_t^w / W_t . \tag{B.2.9}$$

From (B.2.4) and (B.2.7)

$$\upsilon(C_t^w)^{\upsilon\rho-1}(I_t^w)^{\nu\rho}(1-l_t^w)^{(1-\upsilon-\nu)\rho} = [\omega V_{t+1}^w + (1-\omega)V_{t+1}^r]^{\rho-1}\beta(\omega \tfrac{\partial V_{t+1}^w}{\partial A_{t+1}} + (1-\omega)\tfrac{\partial V_{t+1}^r}{\partial A_{t+1}}), \tag{B.2.10}$$

$$\mu = \upsilon(C_t^w)^{\upsilon\rho-1}(I_t^w)^{\nu\rho}(1-l_t^w)^{(1-\upsilon-\nu)\rho}(V_t^w)^{1-\rho}, \tag{B.2.11}$$

Applying the Envelope Theorem with parameter A_t^w,

$$\frac{dV_t^w}{dA_t^w} = \frac{\partial L}{\partial A_t^w} = \mu R_t = \upsilon(C_t^w)^{\upsilon\rho-1}(I_t^w)^{\nu\rho}(1-l_t^w)^{(1-\upsilon-\nu)\rho}(V_t^w)^{1-\rho} R_t . \tag{B.2.12}$$

From (B.2.12), we have

$$\frac{\partial V_{t+1}^w}{\partial A_{t+1}} = \upsilon(C_{t+1}^w)^{\upsilon\rho-1}(I_{t+1}^w)^{\nu\rho}(1-l_{t+1}^w)^{(1-\upsilon-\nu)\rho}(V_{t+1}^w)^{1-\rho} R_{t+1} . \tag{B.2.13}$$

Let us guess the form of V_t^w is analogous to the form of V_t^r

$$V_t^w = (\pi_t)^{-\frac{1}{\rho}}(C_t^w)^{\upsilon}(I_t^w)^{\nu}(1-l_t^w)^{1-\upsilon-\nu}, \tag{B.2.14}$$

and plug (B.2.8), (B.2.9) in (B.2.14), then

$$V_t^w = (\pi_t)^{-\frac{1}{\rho}} C_t^w (\tfrac{\nu}{\upsilon p_t})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t})^{1-\upsilon-\nu}, \tag{B.2.15}$$

$$V_t^r = (\varepsilon_t \pi_t)^{-\frac{1}{\rho}} C_t^r (\tfrac{\nu}{\upsilon p_t^r})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^r})^{(1-\upsilon-\nu)} . \tag{A.2.15}$$

From (A.2.15) and (B.2.15), we have

$$V_{t+1}^{w}=(\pi_{t+1})^{-\frac{1}{\rho}}C_{t+1}^{w}(\tfrac{\nu}{\upsilon p_{t+1}})^{\nu}(\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}})^{1-\upsilon-\nu}, \tag{B.2.16}$$

$$V_{t+1}^{r}=(\varepsilon_{t+1}\pi_{t+1})^{-\frac{1}{\rho}}C_{t+1}^{r}(\tfrac{\nu}{\upsilon p_{t+1}^{r}})^{\nu}(\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^{r}})^{(1-\upsilon-\nu)}. \tag{B.2.17}$$

Using (B.2.8), (B.2.9), and (B.2.16), (B.2.13) can be written as

$$\frac{\partial V_{t+1}^{w}}{\partial A_{t+1}^{w}}=R_{t+1}\upsilon(\tfrac{\nu}{\upsilon p_{t+1}})^{\nu}(\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}})^{(1-\upsilon-\nu)}(\pi_{t+1})^{-\frac{1-\rho}{\rho}}, \tag{B.2.18}$$

$$\frac{\partial V_{t+1}^{r}}{\partial A_{t+1}^{r}}=\tfrac{R_{t+1}}{\gamma}\upsilon(\tfrac{\nu}{\upsilon p_{t+1}})^{\nu}(\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^{r}})^{(1-\upsilon-\nu)}(\varepsilon_{t+1}\pi_{t+1})^{-\frac{1-\rho}{\rho}}, \tag{A.2.17}$$

and placing (A.2.16), (A.2.17), (B.2.16), and (B.2.18) into (B.2.10), then,

$$\begin{aligned}&\upsilon(C_t^{w})^{\rho-1}((\tfrac{1}{p_t})^{\nu}(\tfrac{1}{W_t})^{(1-\upsilon-\nu)})^{\rho}\\&=((\tfrac{1}{p_{t+1}})^{\nu}(\tfrac{1-\upsilon-\nu}{W_{t+1}})^{(1-\upsilon-\nu)})^{\rho-1}[\omega(\pi_{t+1})^{-\frac{1}{\rho}}C_{t+1}^{w}+(1-\omega)(\varepsilon_{t+1}\pi_{t+1})^{-\frac{1}{\rho}}C_{t+1}^{r}(\tfrac{p_{t+1}}{p_{t+1}^{r}})^{\nu}(\tfrac{W_{t+1}}{W_{t+1}^{r}})^{(1-\upsilon-\nu)}]^{\rho-1}\\&\times\beta R_{t+1}\upsilon((\tfrac{1}{p_{t+1}})^{\nu}(\tfrac{1-\upsilon-\nu}{W_{t+1}})^{(1-\upsilon-\nu)})[\omega(\pi_{t+1})^{-\frac{1-\rho}{\rho}}+(1-\omega)\tfrac{1}{\gamma}(\tfrac{p_{t+1}}{p_{t+1}^{r}})^{\nu}(\tfrac{W_{t+1}}{W_{t+1}^{r}})^{(1-\upsilon-\nu)}(\varepsilon_{t+1}\pi_{t+1})^{-\frac{1-\rho}{\rho}}].\end{aligned} \tag{B.2.19}$$

Simplify the (B.2.19)

$$\begin{aligned}(C_t^{w})^{\rho-1}&=((\tfrac{p_t}{p_{t+1}})^{\nu}(\tfrac{W_t}{W_{t+1}})^{(1-\upsilon-\nu)})^{\rho}[\omega C_{t+1}^{w}+(1-\omega)(\varepsilon_{t+1})^{-\frac{1}{\rho}}C_{t+1}^{r}(\tfrac{p_{t+1}}{p_{t+1}^{r}})^{\nu}(\tfrac{W_{t+1}}{W_{t+1}^{r}})^{(1-\upsilon-\nu)}]^{\rho-1}\\&\times\beta R_{t+1}[\omega+(1-\omega)\tfrac{1}{\gamma}(\tfrac{p_{t+1}}{p_{t+1}^{r}})^{\nu}(\tfrac{W_{t+1}}{W_{t+1}^{r}})^{(1-\upsilon-\nu)}(\varepsilon_{t+1})^{-\frac{1-\rho}{\rho}}].\end{aligned} \tag{B.2.20}$$

Let us define that $\eta=\frac{p_{t+1}^{r}}{p_{t+1}}$ and $\xi=\frac{W_{t+1}^{r}}{W_{t+1}}$,

$$\chi=(\tfrac{1}{\eta})^{\nu}(\tfrac{1}{\xi})^{1-\upsilon-\nu},$$

$$\Omega_{t+1}=[\omega+(1-\omega)\tfrac{1}{\gamma}(\varepsilon_{t+1})^{-\frac{1-\rho}{\rho}}\chi]. \tag{B.2.21}$$

Then, (B.2.20) is rewritten as

$$\omega C_{t+1}^{w}+(1-\omega)\chi(\varepsilon_{t+1})^{\frac{\sigma}{1-\sigma}}C_{t+1}^{r}=((\tfrac{P_t}{P_{t+1}})^{\nu}(\tfrac{W_t}{W_{t+1}})^{1-\upsilon-\nu})^{\sigma-1}(R_{t+1}\Omega_{t+1}\beta)^{\sigma}C_t^{w}. \tag{B.2.22}$$

Now, let us guess a consumption solution of the form:

$$C_t^{w}=\pi_t(R_tA_t^{w}+H_t^{w}+S_t^{w}-Z_t^{w}) \tag{B.2.23}$$

Check that

$$C_{t+1}^{w} = \pi_{t+1}(R_{t+1}A_{t+1}^{w} + H_{t+1}^{w} + S_{t+1}^{w} - Z_{t+1}^{w}), \tag{B.2.24}$$

$$C_{t+1}^{r} = \varepsilon_{t+1}\pi_{t+1}(\tfrac{R_{t+1}}{\gamma} A_{t+1}^{r} + H_{t+1}^{r} + S_{t+1}^{r} - Z_{t+1}^{r}). \tag{A.2.22}$$

Applying (B.2.23), (B.2.24) and (A.2.22) to (B.2.22), then we have

$$\begin{aligned}&\pi_{t+1}[\omega(R_{t+1}A_{t+1}^{w} + H_{t+1}^{w} + S_{t+1}^{w} - Z_{t+1}^{w}) + (1-\omega)(\varepsilon_{t+1})^{\frac{1}{1-\sigma}}\chi(\tfrac{R_{t+1}}{\gamma} A_{t+1}^{r} + H_{t+1}^{r} + S_{t+1}^{r} - Z_{t+1}^{r})] \\ &= [(\tfrac{W_t}{W_{t+1}})^{1-\upsilon-\nu}(\tfrac{p_t}{p_{t+1}})^{\nu}]^{\sigma-1}(R_{t+1}\Omega_{t+1}\beta)^{\sigma}\pi_t(R_t A_t^{w} + H_t^{w} + S_t^{w} - Z_t^{w}).\end{aligned} \tag{B.2.25}$$

Now, placing (B.2.15), (B.2.16), (B.2.17) into value function (B.2.1), then

$$\begin{aligned}&[(\pi_t)^{-\frac{1}{\rho}} C_t^{w}(\tfrac{\nu}{\upsilon p_t})^{\nu}(\tfrac{1-\upsilon-\nu}{\upsilon W_t})^{1-\upsilon-\nu}]^{\rho} \\ &= [C_t^{w}(\tfrac{\nu}{\upsilon p_t})^{\nu}(\tfrac{1-\upsilon-\nu}{\upsilon W_t})^{1-\upsilon-\nu}]^{\rho} + \beta[\omega(\pi_{t+1})^{-\frac{1}{\rho}} C_{t+1}^{w}(\tfrac{\nu}{\upsilon p_{t+1}})^{\nu}(\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}})^{1-\upsilon-\nu} + (1-\omega)(\varepsilon_{t+1}\pi_{t+1})^{-\frac{1}{\rho}} C_{t+1}^{r}(\tfrac{\nu}{\upsilon p_{t+1}^{r}})^{\nu}(\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^{r}})^{1-\upsilon-\nu}]^{\rho}.\end{aligned} \tag{B.2.26}$$

which can be written as

$$(\pi_t)^{-1} = 1 + \beta\left\{(\tfrac{P_t}{P_{t+1}})^{\nu}(\tfrac{W_t}{W_{t+1}})^{1-\upsilon-\nu}[\omega C_{t+1}^{w} + (1-\omega)\chi(\varepsilon_{t+1})^{\frac{\sigma}{1-\sigma}} C_{t+1}^{r}]/C_t^{w}\right\}^{\frac{\sigma-1}{\sigma}}(\pi_{t+1})^{-1}, \tag{B.2.27}$$

and placing (B.2.21) into (B.2.27). Hence, we have

$$\pi_t = 1 - [(\tfrac{P_t}{P_{t+1}})^{\nu}(\tfrac{W_t}{W_{t+1}})^{(1-\upsilon-\nu)} R_{t+1}\Omega_{t+1}]^{\sigma-1}\beta^{\sigma}\tfrac{\pi_t}{\pi_{t+1}}. \tag{B.2.28}$$

From (B.2.28), we have

$$\pi_{t+1} = [(\tfrac{P_t}{P_{t+1}})^{\nu}(\tfrac{W_t}{W_{t+1}})^{(1-\upsilon-\nu)} R_{t+1}\Omega_{t+1}]^{\sigma-1}\beta^{\sigma}\pi_t/(1-\pi_t), \tag{B.2.29}$$

and multiply both sides by the same equations as $R_{t+1}\Omega_{t+1}(1-\pi_t)(R_t A_t^{w} + H_t^{w} + S_t^{w} - Z_t^{w})$,

and then we have

$$\begin{aligned}&\pi_{t+1}[R_{t+1}\Omega_{t+1}(1-\pi_t)(R_t A_t^{w} + H_t^{w} + S_t^{w} - Z_t^{w})] \\ &= [(\tfrac{W_t}{W_{t+1}})^{(1-\upsilon-\nu)}(\tfrac{p_t}{p_{t+1}})^{\nu}]^{\sigma-1}(R_{t+1}\Omega_{t+1}\beta)^{\sigma}\pi_t(R_t A_t^{w} + H_t^{w} + S_t^{w} - Z_t^{w}).\end{aligned} \tag{B.2.30}$$

Here, note that the RHS of (B.2.30) is equal to the RHS of (B.2.25). Hence,

$$\begin{aligned}&R_{t+1}\Omega_{t+1}(1-\pi_t)(R_tA_t^w+H_t^w+S_t^w-Z_t^w)\\&=[\omega(R_{t+1}A_{t+1}^w+H_{t+1}^w+S_{t+1}^w-Z_{t+1}^w)+(1-\omega)(\varepsilon_{t+1})^{\frac{1}{1-\sigma}}\chi(\tfrac{R_{t+1}}{\gamma}A_{t+1}^r+H_{t+1}^r+S_{t+1}^r-Z_{t+1}^r)],\end{aligned} \tag{B.2.31}$$

i.e.

$$\begin{aligned}&(1-\pi_t)(R_tA_t^w+H_t^w+S_t^w-Z_t^w)\\&=\tfrac{\omega}{R_{t+1}\Omega_{t+1}}(R_{t+1}A_{t+1}^w+H_{t+1}^w+S_{t+1}^w-Z_{t+1}^w)+\tfrac{(1-\omega)\frac{1}{\gamma}(\varepsilon_{t+1})^{\frac{1}{1-\sigma}}\chi}{R_{t+1}\Omega_{t+1}}(R_{t+1}A_{t+1}^r+H_{t+1}^r+S_{t+1}^r-Z_{t+1}^r)].\end{aligned} \tag{B.2.32}$$

Using (B.2.21), (B.2.32) can be rewritten as

$$\begin{aligned}&(1-\pi_t)(R_tA_t^w+H_t^w+S_t^w-Z_t^w)\\&=\tfrac{1}{R_{t+1}}\tfrac{\omega}{\Omega_{t+1}}(R_{t+1}A_{t+1}^w+H_{t+1}^w+S_{t+1}^w-Z_{t+1}^w)+\tfrac{1}{R_{t+1}}(1-\tfrac{\omega}{\Omega_{t+1}})(R_{t+1}A_{t+1}^r+H_{t+1}^r+S_{t+1}^r-Z_{t+1}^r)].\end{aligned} \tag{B.2.33}$$

It is also possible to express the LHS of (B.2.33) as following:

$$\begin{aligned}&(1-\pi_t)(R_tA_t^w+H_t^w+S_t^w-Z_t^w)\\&=(R_tA_t^w+W_tl_t^w-C_t^w-I_t^wp_t)+(H_t^w-W_tl_t^w)+(S_t^w-0)-(Z_t^w-I_t^wp_t).\end{aligned} \tag{B.2.34}$$

Then, (B.2.35) holds and

$$\begin{aligned}&(R_tA_t^w+W_tl_t^w-C_t^w-I_t^wp_t)+(H_t^w-W_tl_t^w)+(S_t^w-0)-(Z_t^w-I_t^wp_t)\\&=\tfrac{1}{R_{t+1}}\tfrac{\omega}{\Omega_{t+1}}(R_{t+1}A_{t+1}^w+H_{t+1}^w+S_{t+1}^w-Z_{t+1}^w)+\tfrac{1}{R_{t+1}}(1-\tfrac{\omega}{\Omega_{t+1}})(R_{t+1}A_{t+1}^r+H_{t+1}^r+S_{t+1}^r-Z_{t+1}^r)],\end{aligned} \tag{B.2.35}$$

which implies that the worker’ s wealth at the end of time t after cash flows have occurred becomes either the worker's wealth or the retiree's wealth at the end of time t +1 and that the discounted risk-adjusted work probability

$$\frac{1}{R_{t+1}}\frac{\omega}{\Omega_{t+1}} \tag{B.2.36}$$

will play a role in the valuation of worker’ s human wealth(nonfinancial asset) i.e. $H_t^w+S_t^w-Z_t^w$, and the discounted risk-adjusted retirement transition probability

$$\frac{1}{R_{t+1}}(1-\frac{\omega}{\Omega_{t+1}}) \qquad \text{(B.2.37)}$$

will be used when the worker goes from work at time t to retirement at time t+1.

It is obvious from (B.2.2) that $A^w_{t+1} = A^r_{t+1}$,

$$R_t A^w_t + W_t l^w_t - C^w_t - I^w_t p_t = \frac{\omega}{\Omega_{t+1}} A^w_{t+1} + (1-\frac{\omega}{\Omega_{t+1}}) A^r_{t+1}. \qquad \text{(B.2.38)}$$

Thus, we obtain

$$\frac{1}{R_{t+1}}\frac{\omega}{\Omega_{t+1}}(H^w_{t+1} + S^w_{t+1} - Z^w_{t+1}) + \frac{1}{R_{t+1}}(1-\frac{\omega}{\Omega_{t+1}})\gamma(H^r_{t+1} + S^r_{t+1} - Z^r_{t+1}) = (H^w_t + S^w_t - Z^w_t) - W_t l^w_t + I^w_t p_t.$$

(B.2.39)

It is not necessary that the following equations

$$H^w_t = W_t l^w_t + \frac{1}{R_{t+1}}\frac{\omega}{\Omega_{t+1}} H^w_{t+1} + \frac{1}{R_{t+1}}(1-\frac{\omega}{\Omega_{t+1}})\gamma H^r_{t+1}, \qquad \text{(B.2.40)}$$

$$S^w_t = \frac{1}{R_{t+1}}\frac{\omega}{\Omega_{t+1}} S^w_{t+1} + \frac{1}{R_{t+1}}(1-\frac{\omega}{\Omega_{t+1}})\gamma S^r_{t+1}, \qquad \text{(B.2.41)}$$

$$Z^w_t = I^w_t p_t + \frac{1}{R_{t+1}}\frac{\omega}{\Omega_{t+1}} Z^w_{t+1} + \frac{1}{R_{t+1}}(1-\frac{\omega}{\Omega_{t+1}})\gamma Z^r_{t+1}, \qquad \text{(B.2.42)}$$

hold true. Eq. (B.2.39) to derive aggregate demand, supply and steady state endogenous variables. But it is convenient to use equations (B.2.40), (B.2.41) and (B.2.42) for calculation.

To confirm a solution for the value function, conjecture that

$$V^w_t = \Delta^w_t (C^w_t)^{\upsilon} (I^w_t)^{\nu} (1-l^w_t)^{1-\upsilon-\nu} = \Delta^w_t (C^w_t)^{\upsilon} (\frac{\nu}{\upsilon p_t})^{\nu} (\frac{1-\upsilon-\nu}{\upsilon W_t})^{(1-\upsilon-\nu)}. \qquad \text{(B.2.43)}$$

Then, to obtain an expression for Δ^w_t, substitute the conjectured solution for V^w_t into the objective to obtain

$$\Delta_t^w C_t^w (\tfrac{\nu}{\upsilon p_t})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t})^{(1-\upsilon-\nu)}$$

$$= \left[\left\{ C_t^w (\tfrac{\nu}{\upsilon p_t})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t})^{(1-\upsilon-\nu)} \right\}^{\rho} + \beta \left\{ \omega \Delta_{t+1}^w C_{t+1}^w (\tfrac{\nu}{\upsilon p_{t+1}})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}})^{(1-\upsilon-\nu)} + (1-\omega) \Delta_{t+1}^r C_{t+1}^r (\tfrac{\nu}{\upsilon p_{t+1}^r})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^r})^{(1-\upsilon-\nu)} \right\}^{\rho} \right]^{1/\rho}. \tag{B.2.44}$$

Here, placing (B.2.21) into (B.2.44), (B.2.44) can be rewritten as

$$\left\{ \Delta_t^w C_t^w (\tfrac{\nu}{\upsilon p_t})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t})^{(1-\upsilon-\nu)} \right\}^{\rho}$$

$$= \left\{ C_t^w (\tfrac{\nu}{\upsilon p_t})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t})^{(1-\upsilon-\nu)} \right\}^{\rho} + \beta \left[\Delta_{t+1}^w \left\{ (\tfrac{W_t}{W_{t+1}})^{(1-\upsilon-\nu)\rho} (\tfrac{p_t}{p_{t+1}})^{\nu\rho} R_{t+1} \Omega_{t+1} \beta \right\}^{\sigma} C_t^w (\tfrac{\nu}{\upsilon p_t})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t})^{(1-\upsilon-\nu)} \right]^{\rho}. \tag{B.2.45}$$

Thus, applying that $\sigma\rho + 1 = \sigma$ and $\sigma\rho = \sigma - 1$, we have

$$(\Delta_t^w)^{\rho} = 1 + [(\tfrac{W_t}{W_{t+1}})^{(1-\upsilon-\nu)} (\tfrac{p_t}{p_{t+1}})^{\nu} R_{t+1} \Omega_{t+1}]^{\sigma-1} \beta^{\sigma} (\Delta_{t+1}^w)^{\rho}. \tag{B.2.46}$$

Now, checking that (B.2.46) is identical to (B.2.47), we obtain

$$\Delta_t^w = (\pi_t)^{-\frac{1}{\rho}}. \tag{B.2.47}$$

Thus,

$$V_t^w = (\pi_t)^{-\frac{1}{\rho}} C_t^w (\tfrac{\nu}{\upsilon p_t})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t})^{(1-\upsilon-\nu)}. \tag{B.2.48}$$

<Appendix C> Life insurance model

1) Retiree/Dependent decision problems

Maximize

$$V_t^j = \left\{ [(C_t^j)^{\upsilon} (I_t^j)^{\nu} (1-l_t^j)^{1-\upsilon-\nu}]^{\rho} + \beta\gamma_j (V_{t+1}^j)^{\rho} \right\}^{1/\rho}, \qquad \text{(C.1.1)}$$

subject to

$$A_{t+1}^j = \tfrac{R_t}{\gamma_j} A_t^j + W_t^j l_t^j + E_t^j - C_t^j - I_t^j p_t^j, \qquad \text{(C.1.2)}$$

where $j = r$ (retiree) or d (dependent)

Using Lagrange multiplier μ, the optimization problem can be rewritten as

$$L = V_t^j - \mu(A_{t+1}^j - \tfrac{R_t}{\gamma_j} A_t^j - W_t^j l_t^j - E_t^j + C_t^j + I_t^j p_t^j). \qquad \text{(C.1.3)}$$

Let us partially differentiate L with respect to C_t^j, I_t^j, l_t^j and A_{t+1}^j. From the four partial derivatives, i.e.

$$\frac{\partial L}{\partial C_t^j} = (V_t^j)^{1-\rho} \upsilon (C_t^j)^{\upsilon\rho-1} (I_t^j)^{\nu\rho} (1-l_t^j)^{(1-\upsilon-\nu)\rho} - \mu = 0, \qquad \text{(C.1.4)}$$

$$\frac{\partial L}{\partial I_t^j} = (V_t^j)^{1-\rho} \nu (C_t^j)^{\upsilon\rho-1} (I_t^j)^{\nu\rho-1} (1-l_t^j)^{(1-\upsilon-\nu)\rho} - \mu p_t^j = 0, \qquad \text{(C.1.5)}$$

$$\frac{\partial L}{\partial l_t^j} = -(V_t^j)^{1-\rho} (1-\upsilon-\nu)(C_t^j)^{\upsilon\rho} (I_t^j)^{\nu\rho} (1-l_t^j)^{(1-\upsilon-\nu)\rho-1} + \mu W_t^j = 0, \qquad \text{(C.1.6)}$$

$$\frac{\partial L}{\partial A_{t+1}^j} = (V_t^j)^{1-\rho} \beta\gamma_j \tfrac{\partial V_{t+1}^j}{\partial A_{t+1}^j} (V_{t+1}^j)^{\rho-1} - \mu = 0, \qquad \text{(C.1.7)}$$

we have

$$I_t^j p_t^j = \tfrac{\nu}{\upsilon} C_t^j, \qquad \text{(C.1.8)}$$

$$1-l_t^j = \tfrac{1-\upsilon-\nu}{\upsilon} C_t^j / W_t^j \,. \qquad \text{(C.1.9)}$$

From (C.1.4) and (C.1.7)

$$\upsilon (C_t^j)^{\upsilon\rho-1} (I_t^j)^{\nu\rho} (1-l_t^j)^{(1-\upsilon-\nu)\rho} = \beta \gamma_j \tfrac{\partial V_{t+1}^j}{\partial A_{t+1}^j} (V_{t+1}^j)^{\rho-1} \,, \qquad \text{(C.1.10)}$$

$$\mu = \upsilon (C_t^j)^{\upsilon\rho-1} (I_t^j)^{\nu\rho} (1-l_t^j)^{(1-\upsilon-\nu)\rho} (V_t^j)^{1-\rho} \,. \qquad \text{(C.1.11)}$$

Applying the Envelope Theorem with parameter A_t^j,

$$\frac{dV_t^j}{dA_t^j} = \frac{\partial L}{\partial A_t^j} = \mu \tfrac{R_t}{\gamma_j} = \tfrac{R_t}{\gamma_j} \upsilon (C_t^j)^{\upsilon\rho-1} (I_t^j)^{\nu\rho} (1-l_t^j)^{(1-\upsilon-\nu)\rho} (V_t^j)^{1-\rho} \qquad \text{(C.1.12)}$$

From (C.1.12), we have

$$\frac{\partial V_{t+1}^j}{\partial A_{t+1}^j} = \tfrac{R_{t+1}}{\gamma_j} \upsilon (C_{t+1}^j)^{\upsilon\rho-1} (I_{t+1}^j)^{\nu\rho} (1-l_{t+1}^j)^{(1-\upsilon-\nu)\rho} (V_{t+1}^j)^{1-\rho} \,. \qquad \text{(C.1.13)}$$

Let us guess the form of V_t^j

$$V_t^j = (\varepsilon_t^j \pi_t)^{-\frac{1}{\rho}} (C_t^j)^{\upsilon} (I_t^j)^{\nu} (1-l_t^j)^{1-\upsilon-\nu} \,, \qquad \text{(C.1.14)}$$

and plug (C.1.8), (C.1.9) in (C.1.14), then

$$V_t^j = (\varepsilon_t^j \pi_t)^{-\frac{1}{\rho}} C_t^j (\tfrac{\nu}{\upsilon p_t^j})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^j})^{(1-\upsilon-\nu)} \,. \qquad \text{(C.1.15)}$$

From (C.1.15), we have

$$V_{t+1}^j = (\varepsilon_{t+1}^j \pi_{t+1})^{-\frac{1}{\rho}} C_{t+1}^j (\tfrac{\nu}{\upsilon p_{t+1}^j})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^j})^{(1-\upsilon-\nu)} \,. \qquad \text{(C.1.16)}$$

Placing (C.1.8), (C.1.9), and (C.1.16) into (C.1.13)

$$\frac{\partial V_{t+1}^j}{\partial A_{t+1}^j} = \tfrac{R_{t+1}}{\gamma_j} \upsilon (\tfrac{\nu}{\upsilon p_{t+1}})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^j})^{(1-\upsilon-\nu)} (\varepsilon_{t+1}^j \pi_{t+1})^{-\frac{1-\rho}{\rho}} \qquad \text{(C.1.17)}$$

Substituting (C.1.16), (C.1.17) into (C.1.10)

$$\upsilon (C_t^j)^{\upsilon\rho-1} (I_t^j)^{\nu\rho} (1-l_t^j)^{(1-\upsilon-\nu)\rho} = \beta R_{t+1} \upsilon (\tfrac{\nu}{\upsilon p_{t+1}^j})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^j})^{1-\upsilon-\nu} [C_{t+1}^j (\tfrac{\nu}{\upsilon p_{t+1}^j})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^j})^{1-\upsilon-\nu}]^{\rho-1}, \qquad \text{(C.1.18)}$$

which is rewritten as

$$(C_{t+1}^j)^{1-\rho} = (\tfrac{W_t^j}{W_{t+1}^j})^{(1-\upsilon-\nu)\rho} (\tfrac{p_t^j}{p_{t+1}^j})^{\nu\rho} R_{t+1}\beta (C_t^j)^{1-\rho}, \qquad (C.1.19)$$

$$C_{t+1}^j = [(\tfrac{W_t^j}{W_{t+1}^j})^{(1-\upsilon-\nu)\rho} (\tfrac{p_t^j}{p_{t+1}^j})^{\nu\rho} R_{t+1}\beta]^{\sigma} C_t^j, \qquad (C.1.20)$$

where $\sigma = 1/(1-\rho)$ and $W_t^j = \xi_j W_t$.

Next, let us guess a solution of the form:

$$C_t^j = \varepsilon_t^j \pi_t (\tfrac{R_t}{\gamma_j} A_t^j + H_t^j + S_t^j - Z_t^j). \qquad (C.1.21)$$

Check that

$$C_{t+1}^j = \varepsilon_{t+1}^j \pi_{t+1} (\tfrac{R_{t+1}}{\gamma_j} A_{t+1}^j + H_{t+1}^j + S_{t+1}^j - Z_{t+1}^j). \qquad (C.1.22)$$

Placing (C.1.21), (C.1.22) into (C.1.20), we can obtain

$$\varepsilon_{t+1}^j \pi_{t+1} [\tfrac{R_{t+1}}{\gamma_j} A_{t+1}^j + (H_{t+1}^j + S_{t+1}^j - Z_{t+1}^j)] = [(\tfrac{W_t^j}{W_{t+1}^j})^{(1-\upsilon-\nu)\rho} (\tfrac{p_t^j}{p_{t+1}^j})^{\nu\rho} R_{t+1}\beta]^{\sigma} \varepsilon_t^j \pi_t [\tfrac{R_t}{\gamma_j} A_t^j + (H_t^j + S_t^j - Z_t^j)].$$

(C.1.23)

Now, placing (C.1.15), (C.1.16) into value function (C.1.1), then

$$[(\varepsilon_t^j \pi_t)^{-\frac{1}{\rho}} C_t^j (\tfrac{\nu}{\upsilon p_t^j})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^j})^{1-\upsilon-\nu}]^{\rho} = [C_t^j (\tfrac{\nu}{\upsilon p_t^j})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^j})^{1-\upsilon-\nu}]^{\rho} + \beta\gamma_j [(\varepsilon_{t+1}^j \pi_{t+1})^{-\frac{1}{\rho}} C_{t+1}^j (\tfrac{\nu}{\upsilon p_{t+1}^j})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^j})^{1-\upsilon-\nu}]^{\rho}.$$

(C.1.24)

Replacing C_{t+1}^j to the form of C_t^j using the relation (C.1.20)

$$\begin{aligned} &[(\varepsilon_t^j \pi_t)^{-\frac{1}{\rho}} C_t^j (\tfrac{\nu}{\upsilon p_t^j})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^j})^{1-\upsilon-\nu}]^{\rho} \\ &= [C_t^j (\tfrac{\nu}{\upsilon p_t^j})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^j})^{1-\upsilon-\nu}]^{\rho} + \beta\gamma_j [(\varepsilon_{t+1}^j \pi_{t+1})^{-\frac{1}{\rho}} ((\tfrac{W_t^j}{W_{t+1}^j})^{(1-\upsilon-\nu)\rho} (\tfrac{p_t^j}{p_{t+1}^j})^{\nu\rho} R_{t+1}\beta)^{\sigma} C_t^j (\tfrac{\nu}{\upsilon p_{t+1}^j})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^j})^{1-\upsilon-\nu}]^{\rho}, \end{aligned}$$

(C.1.25)

which can be written as

$$(\varepsilon_t^j \pi_t)^{-1} = 1 + [(\tfrac{W_t^j}{W_{t+1}^j})^{1-\upsilon-\nu} (\tfrac{p_t^j}{p_{t+1}^j})^{\nu} R_{t+1}]^{\sigma-1} \beta^{\sigma} \gamma_j (\varepsilon_{t+1}^j \pi_{t+1})^{-1}, \tag{C.1.26}$$

$$\varepsilon_t^j \pi_t = 1 - [(\tfrac{W_t^j}{W_{t+1}^j})^{1-\upsilon-\nu} (\tfrac{p_t^j}{p_{t+1}^j})^{\nu} R_{t+1}]^{\sigma-1} \beta^{\sigma} \gamma_j \tfrac{\varepsilon_t^j \pi_t}{\varepsilon_{t+1}^j \pi_{t+1}}. \tag{C.1.27}$$

From (C.1.27), we have

$$\varepsilon_t^j \pi_t = [(\tfrac{W_t^j}{W_{t+1}^j})^{1-\upsilon-\nu} (\tfrac{p_t^j}{p_{t+1}^j})^{\nu} R_{t+1}]^{\sigma-1} \beta^{\sigma} \gamma_j \tfrac{\varepsilon_t^j \pi_t}{(1-\varepsilon_t^j \pi_t)}, \tag{C.1.28}$$

and multiply both sides by the same equations as $R_{t+1}(1-\varepsilon_t^j \pi_t)(\frac{R_t}{\gamma_j} A_t^j + H_t^j + S_t^j - Z_t^j)$,

and then we have

$$\begin{aligned} &\varepsilon_{t+1}^j \pi_{t+1} [R_{t+1}(1-\varepsilon_t^j \pi_t)(\tfrac{R_t}{\gamma_j} A_t^j + H_t^j + S_t^j - Z_t^j)] \\ &= [(\tfrac{W_t^j}{W_{t+1}^j})^{1-\upsilon-\nu} (\tfrac{p_t^j}{p_{t+1}^j})^{\nu}]^{\sigma-1} (R_{t+1}\beta)^{\sigma} \gamma_j \varepsilon_t^j \pi_t (\tfrac{R_t}{\gamma_j} A_t^j + H_t^j + S_t^j - Z_t^j). \end{aligned} \tag{C.1.29}$$

Here, note that the RHS of (C.1.29) is equal to the product of γ_j and the RHS of (C.1.23). Hence,

$$R_{t+1}(1-\varepsilon_t^j \pi_t)(\tfrac{R_t}{\gamma_j} A_t^j + H_t^j + S_t^j - Z_t^j) = \gamma_j (\tfrac{R_{t+1}}{\gamma_j} A_{t+1}^j + H_{t+1}^j + S_{t+1}^j - Z_{t+1}^j), \tag{C.1.30}$$

i.e.

$$(1-\varepsilon_t^j \pi_t)(\tfrac{R_t}{\gamma_j} A_t^j + H_t^j + S_t^j - Z_t^j) = A_{t+1}^j + \tfrac{\gamma_j}{R_{t+1}} (H_{t+1}^j + S_{t+1}^j - Z_{t+1}^j). \tag{C.1.31}$$

It is also possible to express the LHS of (C.1.31) as (C.1.32).

$$\begin{aligned} &(1-\varepsilon_t^j \pi_t)(\tfrac{R_t}{\gamma_j} A_t^j + H_t^j + S_t^j - Z_t^j) \\ &= (\tfrac{R_t}{\gamma_j} A_t^j + W_t^j l_t^j + E_t^j - C_t^j - I_t^j p_t^j) + (H_t^j - W_t^j l_t^j) + (S_t^j - E_t^j) - (Z_t^j - I_t^j p_t^j). \end{aligned} \tag{C.1.32}$$

Then, from (C.1.31) and (C.1.32), following equation holds

$$\begin{aligned} &A_{t+1}^j + \tfrac{\gamma_j}{R_{t+1}} (H_{t+1}^j + S_{t+1}^j - Z_{t+1}^j) \\ &= (\tfrac{R_t}{\gamma_j} A_t^j + W_t^j l_t^j + E_t^j - C_t^j - I_t^j p_t^j) + (H_t^j - W_t^j l_t^j) + (S_t^j - E_t^j) - (Z_t^j - I_t^j p_t^j). \end{aligned} \tag{C.1.33}$$

It is obvious that from (C.1.2)

$$\tfrac{R_t}{\gamma_j} A_t^j + W_t^j l_t^j + E_t^j - C_t^j - I_t^j p_t^j = A_{t+1}^j . \tag{C.1.34}$$

Thus, we obtain

$$\tfrac{\gamma_j}{R_{t+1}} (H_{t+1}^j + S_{t+1}^j - Z_{t+1}^j) = (H_t^j - W_t^j l_t^j) + (S_t^j - E_t^j) - (Z_t^j - I_t^j p_t^j) , \tag{C.1.35}$$

i.e.

$$H_t^j + S_t^j - Z_t^j = W_t^j l_t^j + E_t^j - I_t^j p_t^j + \tfrac{\gamma_j}{R_{t+1}} (H_{t+1}^j + S_{t+1}^j - Z_{t+1}^j) . \tag{C.1.36}$$

It is not necessary that the following equations

$$H_t^j = W_t^j l_t^j + \tfrac{\gamma_j}{R_{t+1}} H_{t+1}^j , \tag{C.1.37}$$

$$S_t^j = E_t^j + \tfrac{\gamma_j}{R_{t+1}} S_{t+1}^j , \tag{C.1.38}$$

$$Z_t^j = I_t^j p_t^j + \tfrac{\gamma_j}{R_{t+1}} Z_{t+1}^j , \tag{C.1.39}$$

hold true. Eq.(C.1.36) to derive aggregate demand, supply and steady state endogenous variables. But, it is convenient to use equations (C.1.37), (C.1.38) and (C.1.39) for calculation.

To confirm a solution for the value function, conjecture that

$$V_t^j = \Delta_t^j (C_t^j)^{\upsilon} (I_t^j)^{\nu} (1 - l_t^j)^{1-\upsilon-\nu} = \Delta_t^j (C_t^j)^{\upsilon} (\tfrac{\nu}{\upsilon p_t^j})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^j})^{(1-\upsilon-\nu)} . \tag{C.1.40}$$

Then, to obtain an expression for Δ_t^j, substitute the conjectured solution for V_t^j into the objective to obtain

$$\Delta_t^j C_t^j (\tfrac{\nu}{\upsilon p_t^j})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^j})^{(1-\upsilon-\nu)} = \left[\left\{ C_t^j (\tfrac{\nu}{\upsilon p_t^j})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^j})^{(1-\upsilon-\nu)} \right\}^{\rho} + \beta \gamma_j \left\{ \Delta_{t+1}^j C_{t+1}^j (\tfrac{\nu}{\upsilon p_{t+1}^j})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^j})^{(1-\upsilon-\nu)} \right\}^{\rho} \right]^{1/\rho} . \tag{C.1.41}$$

Here, placing (C.1.20) into (C.1.41), (C.1.41) can be rewritten as

$$\left\{\Delta_t^j C_t^j (\tfrac{\nu}{\upsilon p_t^j})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^j})^{(1-\upsilon-\nu)}\right\}^{\rho}$$
$$=\left\{C_t^j (\tfrac{\nu}{\upsilon p_t^j})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^j})^{(1-\upsilon-\nu)}\right\}^{\rho} + \beta\gamma_j \left[\Delta_{t+1}^j \left\{(\tfrac{W_t^j}{W_{t+1}^j})^{(1-\upsilon-\nu)\rho} (\tfrac{p_t^j}{p_{t+1}^j})^{\nu\rho} R_{t+1}\beta\right\}^{\sigma} C_t^j (\tfrac{\nu}{\upsilon p_t^j})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^j})^{(1-\upsilon-\nu)}\right]^{\rho}.$$

(C.1.42)

Thus, applying that $\sigma\rho+1=\sigma$ and $\sigma\rho=\sigma-1$, we have

$$(\Delta_t^j)^{\rho} = 1+[(\tfrac{W_t^j}{W_{t+1}^j})^{(1-\upsilon-\nu)} (\tfrac{p_t^j}{p_{t+1}^j})^{\nu} R_{t+1}]^{\sigma-1} \beta^{\sigma}\gamma_j (\Delta_{t+1}^j)^{\rho}. \qquad \text{(C.1.43)}$$

Now, checking that (C.1.43) is identical to (C.1.26), we obtain

$$\Delta_t^j = (\varepsilon_t^j \pi_t)^{-\frac{1}{\rho}}. \qquad \text{(C.1.44)}$$

Thus,

$$V_t^j = \left(\varepsilon_t^j \pi_t\right)^{-\frac{1}{\rho}} C_t^j (\tfrac{\nu}{\upsilon p_t^j})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^j})^{(1-\upsilon-\nu)}. \qquad \text{(C.1.45)}$$

2) Worker decision problems

Maximize

$$V_t^w = \left\{[(C_t^w)^{\upsilon}(I_t^w)^{\nu}(1-l_t^w)^{1-\upsilon-\nu}]^{\rho} + \beta\gamma_w[\omega V_{t+1}^w + (1-\omega)V_{t+1}^r]^{\rho}\right\}^{1/\rho}, \qquad \text{(C.2.1)}$$

subject to

$$A_{t+1}^w = R_t A_t^w + W_t\, l_t^w - C_t^w - I_t^w p_t\,, \qquad \text{(C.2.2)}$$

where p_t is risk probability and W_t is after-tax wage.

Using Lagrange multiplier μ, the optimization problem can be written as

$$L = V_t^w - \mu(A_{t+1}^w - R_t A_t^w - W_t\, l_t^w + C_t^w + I_t^w p_t\,). \qquad \text{(C.2.3)}$$

Let us partially differentiate L with respect to C_t^w, I_t^w, l_t^w and A_{t+1}^w. From the four partial derivatives, i.e.

$$\frac{\partial L}{\partial C_t^w}=(V_t^w)^{1-\rho}\upsilon(C_t^w)^{\upsilon\rho-1}(I_t^w)^{\nu\rho}(1-l_t^w)^{(1-\upsilon-\nu)\rho}-\mu=0\,, \tag{C.2.4}$$

$$\frac{\partial L}{\partial I_t^w}=(V_t^w)^{1-\rho}\nu(C_t^w)^{\upsilon\rho-1}(I_t^w)^{\nu\rho-1}(1-l_t^w)^{(1-\upsilon-\nu)\rho}-\mu p_t=0\,, \tag{C.2.5}$$

$$\frac{\partial L}{\partial l_t^w}=-(V_t^w)^{1-\rho}(1-\upsilon-\nu)(C_t^w)^{\upsilon\rho}(I_t^w)^{\nu\rho}(1-l_t^w)^{(1-\upsilon-\nu)\rho-1}+\mu W_t=0\,, \tag{C.2.6}$$

$$\frac{\partial L}{\partial A_{t+1}}=(V_t^w)^{1-\rho}[\omega V_{t+1}^w+(1-\omega)V_{t+1}^r]^{\rho-1}\beta\gamma_w[\omega\tfrac{\partial V_{t+1}^w}{\partial A_{t+1}}+(1-\omega)\tfrac{\partial V_{t+1}^r}{\partial A_{t+1}}]-\mu=0\,, \tag{C.2.7}$$

we have

$$I_t^w p_t=\tfrac{\nu}{\upsilon}C_t^w\,, \tag{C.2.8}$$

$$1-l_t^w=\tfrac{1-\upsilon-\nu}{\upsilon}C_t^w/W_t\,. \tag{C.2.9}$$

From (C.2.4) and (C.2.7)

$$\upsilon(C_t^w)^{\upsilon\rho-1}(I_t^w)^{\nu\rho}(1-l_t^w)^{(1-\upsilon-\nu)\rho}=[\omega V_{t+1}^w+(1-\omega)V_{t+1}^r]^{\rho-1}\beta\gamma_w(\omega\tfrac{\partial V_{t+1}^w}{\partial A_{t+1}}+(1-\omega)\tfrac{\partial V_{t+1}^r}{\partial A_{t+1}})\,, \tag{C.2.10}$$

$$\mu=\upsilon(C_t^w)^{\upsilon\rho-1}(I_t^w)^{\nu\rho}(1-l_t^w)^{(1-\upsilon-\nu)\rho}(V_t^w)^{1-\rho}\,. \tag{C.2.11}$$

Applying the Envelope Theorem with parameter A_t^w,

$$\frac{dV_t^w}{dA_t^w}=\frac{\partial L}{\partial A_t^w}=\mu R_t=\upsilon(C_t^w)^{\upsilon\rho-1}(I_t^w)^{\nu\rho}(1-l_t^w)^{(1-\upsilon-\nu)\rho}(V_t^w)^{1-\rho}R_t\,. \tag{C.2.12}$$

From (C.2.12), we have

$$\frac{\partial V_{t+1}^w}{\partial A_{t+1}}=\upsilon(C_{t+1}^w)^{\upsilon\rho-1}(I_{t+1}^w)^{\nu\rho}(1-l_{t+1}^w)^{(1-\upsilon-\nu)\rho}(V_{t+1}^w)^{1-\rho}R_{t+1}\,. \tag{C.2.13}$$

Let us guess the form of V_t^w is analogous to the form of V_t^r

$$V_t^w = (\pi_t)^{-\frac{1}{\rho}} (C_t^w)^{\upsilon} (I_t^w)^{\nu} (1-l_t^w)^{1-\upsilon-\nu}, \tag{C.2.14}$$

and plug (C.2.8), (C.2.9) in (C.2.14), then

$$V_t^w = (\pi_t)^{-\frac{1}{\rho}} C_t^w (\tfrac{\nu}{\upsilon p_t})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t})^{1-\upsilon-\nu}, \tag{C.2.15}$$

$$V_t^r = (\varepsilon_t^r \pi_t)^{-\frac{1}{\rho}} C_t^r (\tfrac{\nu}{\upsilon p_t^r})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_t^r})^{(1-\upsilon-\nu)}. \tag{C.1.15}$$

From (C.2.45) and (C.1.16), we have

$$V_{t+1}^w = (\pi_{t+1})^{-\frac{1}{\rho}} C_{t+1}^w (\tfrac{\nu}{\upsilon p_{t+1}})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}})^{1-\upsilon-\nu}, \tag{C.2.16}$$

$$V_{t+1}^r = (\varepsilon_{t+1}^r \pi_{t+1})^{-\frac{1}{\rho}} C_{t+1}^r (\tfrac{\nu}{\upsilon p_{t+1}^r})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^r})^{(1-\upsilon-\nu)}. \tag{C.1.16}$$

Using (C.2.8), (C.2.9), and (C.2.16), (C.2.13) can be written as

$$\frac{\partial V_{t+1}^w}{\partial A_{t+1}^w} = R_{t+1} \upsilon (\tfrac{\nu}{\upsilon p_{t+1}})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}})^{(1-\upsilon-\nu)} (\pi_{t+1})^{-\frac{1-\rho}{\rho}} \quad , \tag{C.2.17}$$

$$\frac{\partial V_{t+1}^r}{\partial A_{t+1}^r} = \tfrac{R_{t+1}}{\gamma_r} \upsilon (\tfrac{\nu}{\upsilon p_{t+1}})^{\nu} (\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^r})^{(1-\upsilon-\nu)} (\varepsilon_{t+1}^r \pi_{t+1})^{-\frac{1-\rho}{\rho}}, \tag{C.1.17}$$

and placing (C.1.16), (C.1.17), (C.2.16), and (C.2.17) into (C.2.10), then,

$$\begin{aligned} &\upsilon (C_t^w)^{\rho-1} ((\tfrac{1}{p_t})^{\nu} (\tfrac{1}{W_t})^{(1-\upsilon-\nu)})^{\rho} \\ &= ((\tfrac{1}{p_{t+1}})^{\nu} (\tfrac{1-\upsilon-\nu}{W_{t+1}})^{(1-\upsilon-\nu)})^{\rho-1} [\omega (\pi_{t+1})^{-\frac{1}{\rho}} C_{t+1}^w + (1-\omega)(\varepsilon_{t+1}\pi_{t+1})^{-\frac{1}{\rho}} C_{t+1}^r (\tfrac{p_{t+1}}{p_{t+1}^r})^{\nu} (\tfrac{W_{t+1}}{W_{t+1}^r})^{(1-\upsilon-\nu)}]^{\rho-1} \\ &\times \beta R_{t+1} \gamma_w \upsilon ((\tfrac{1}{p_{t+1}})^{\nu} (\tfrac{1-\upsilon-\nu}{W_{t+1}})^{(1-\upsilon-\nu)}) [\omega (\pi_{t+1})^{-\frac{1-\rho}{\rho}} + (1-\omega) \tfrac{1}{\gamma_r} (\tfrac{p_{t+1}}{p_{t+1}^r})^{\nu} (\tfrac{W_{t+1}}{W_{t+1}^r})^{(1-\upsilon-\nu)} (\varepsilon_{t+1}\pi_{t+1})^{-\frac{1-\rho}{\rho}}]. \end{aligned} \tag{C.2.18}$$

Simplify the (C.2.18)

$$\begin{aligned} (C_t^w)^{\rho-1} &= ((\tfrac{p_t}{p_{t+1}})^{\nu} (\tfrac{W_t}{W_{t+1}})^{(1-\upsilon-\nu)})^{\rho} [\omega C_{t+1}^w + (1-\omega)(\varepsilon_{t+1})^{-\frac{1}{\rho}} C_{t+1}^r (\tfrac{p_{t+1}}{p_{t+1}^r})^{\nu} (\tfrac{W_{t+1}}{W_{t+1}^r})^{(1-\upsilon-\nu)}]^{\rho-1} \\ &\times \beta R_{t+1} \gamma_w [\omega + (1-\omega) \tfrac{1}{\gamma_r} (\tfrac{p_{t+1}}{p_{t+1}^r})^{\nu} (\tfrac{W_{t+1}}{W_{t+1}^r})^{(1-\upsilon-\nu)} (\varepsilon_{t+1})^{-\frac{1-\rho}{\rho}}] \end{aligned} \tag{C.2.19}$$

Let us define that $\eta_r = \frac{P^r_{t+1}}{P_{t+1}}$ and $\xi_r = \frac{W^r_{t+1}}{W_{t+1}}$,

$$\chi_r = (\tfrac{1}{\eta_r})^{\nu}(\tfrac{1}{\xi_r})^{1-\upsilon-\nu}$$

$$\Omega_{t+1} = \gamma_w[\omega + (1-\omega)\tfrac{1}{\gamma_r}(\varepsilon^r_{t+1})^{-\frac{1-\rho}{\rho}}\chi_r]. \tag{C.2.20}$$

Then, (C.2.19) is rewritten as

$$\omega C^w_{t+1} + (1-\omega)\chi_r(\varepsilon_{t+1})^{\frac{\sigma}{1-\sigma}}C^r_{t+1} = ((\tfrac{p_t}{p_{t+1}})^{\nu}(\tfrac{W_t}{W_{t+1}})^{1-\upsilon-\nu})^{\sigma-1}(R_{t+1}\Omega_{t+1}\beta)^{\sigma}C^w_t. \tag{C.2.21}$$

Now, let us guess a consumption solution of the form:

$$C^w_t = \pi_t(R_t A^w_t + H^w_t + S^w_t - Z^w_t). \tag{C.2.22}$$

Check that

$$C^w_{t+1} = \pi_{t+1}(R_{t+1}A^w_{t+1} + H^w_{t+1} + S^w_{t+1} - Z^w_{t+1}), \tag{C.2.23}$$

$$C^r_{t+1} = \varepsilon^r_{t+1}\pi_{t+1}(\tfrac{R_{t+1}}{\gamma_r}A^r_{t+1} + H^r_{t+1} + S^r_{t+1} - Z^r_{t+1}). \tag{C.1.22}$$

Applying (C.2.22), (C.2.23) and (C.1.22) to (C.2.22), then we have

$$\begin{aligned}&\pi_{t+1}[\omega(R_{t+1}A^w_{t+1} + H^w_{t+1} + S^w_{t+1} - Z^w_{t+1}) + (1-\omega)(\varepsilon^r_{t+1})^{\frac{1}{1-\sigma}}\chi_r(\tfrac{R_{t+1}}{\gamma_r}A^r_{t+1} + H^r_{t+1} + S^r_{t+1} - Z^r_{t+1})]\\ &= [(\tfrac{W_t}{W_{t+1}})^{1-\upsilon-\nu}(\tfrac{p_t}{p_{t+1}})^{\nu}]^{\sigma-1}(R_{t+1}\Omega_{t+1}\beta)^{\sigma}\pi_t(R_t A^w_t + H^w_t + S^w_t - Z^w_t).\end{aligned} \tag{C.2.24}$$

Now, placing (C.2.15), (C.2.16), (C.1.16) into value function (C.2.1), then

$$\begin{aligned}&[(\pi_t)^{-\frac{1}{\rho}}C^w_t(\tfrac{\nu}{\upsilon p_t})^{\nu}(\tfrac{1-\upsilon-\nu}{\upsilon W_t})^{1-\upsilon-\nu}]^{\rho} = [C^w_t(\tfrac{\nu}{\upsilon p_t})^{\nu}(\tfrac{1-\upsilon-\nu}{\upsilon W_t})^{1-\upsilon-\nu}]^{\rho}\\ &+\beta\gamma_w[\omega(\pi_{t+1})^{-\frac{1}{\rho}}C^w_{t+1}(\tfrac{\nu}{\upsilon p_{t+1}})^{\nu}(\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}})^{1-\upsilon-\nu} + (1-\omega)(\varepsilon^r_{t+1}\pi_{t+1})^{-\frac{1}{\rho}}C^r_{t+1}(\tfrac{\nu}{\upsilon p^r_{t+1}})^{\nu}(\tfrac{1-\upsilon-\nu}{\upsilon W^r_{t+1}})^{1-\upsilon-\nu}]^{\rho},\end{aligned} \tag{C.2.25}$$

which can be written as

$$(\pi_t)^{-1} = 1 + \beta\gamma_w\left\{(\tfrac{p_t}{p_{t+1}})^{\nu}(\tfrac{W_t}{W_{t+1}})^{1-\upsilon-\nu}[\omega C^w_{t+1} + (1-\omega)\chi_r(\varepsilon^r_{t+1})^{\frac{\sigma}{1-\sigma}}C^r_{t+1}]/C^w_t\right\}^{\frac{\sigma-1}{\sigma}}(\pi_{t+1})^{-1}, \tag{C.2.26}$$

and placing (C.2.20) into (C.2.26). Hence, we have

$$\pi_t = 1-[(\tfrac{p_t}{p_{t+1}})^{\nu}(\tfrac{W_t}{W_{t+1}})^{(1-\upsilon-\nu)} R_{t+1}\Omega_{t+1}]^{\sigma-1}\beta^{\sigma}\gamma_w \tfrac{\pi_t}{\pi_{t+1}}. \tag{C.2.27}$$

From (C.2.27), we have

$$\pi_{t+1} = [(\tfrac{p_t}{p_{t+1}})^{\nu}(\tfrac{W_t}{W_{t+1}})^{(1-\upsilon-\nu)} R_{t+1}\Omega_{t+1}]^{\sigma-1}\beta^{\sigma}\gamma_w\pi_t/(1-\pi_t), \tag{C.2.28}$$

and multiply both sides by the same equations as $R_{t+1}\Omega_{t+1}(1-\pi_t)(R_tA_t^w+H_t^w+S_t^w-Z_t^w)$, and then we have

$$\begin{aligned}&\pi_{t+1}[R_{t+1}\Omega_{t+1}(1-\pi_t)(R_tA_t^w+H_t^w+S_t^w-Z_t^w)]\\&=[\left(\tfrac{W_t}{W_{t+1}}\right)^{(1-\upsilon-\nu)}\left(\tfrac{p_t}{p_{t+1}}\right)^{\nu}]^{\sigma-1}(R_{t+1}\Omega_{t+1}\beta)^{\sigma}\gamma_w\pi_t(R_tA_t^w+H_t^w+S_t^w-Z_t^w).\end{aligned} \tag{C.2.29}$$

Here, note that the RHS of (C.2.29) is equal to the product of γ_w and the RHS of (C.2.24). Hence,

$$\begin{aligned}&R_{t+1}\Omega_{t+1}(1-\pi_t)(R_tA_t^w+H_t^w+S_t^w-Z_t^w)\\&=\gamma_w[\omega(R_{t+1}A_{t+1}^w+H_{t+1}^w+S_{t+1}^w-Z_{t+1}^w)+(1-\omega)(\varepsilon_{t+1}^r)^{\frac{1}{1-\sigma}}\chi_r(\tfrac{R_{t+1}}{\gamma_r}A_{t+1}^r+H_{t+1}^r+S_{t+1}^r-Z_{t+1}^r)],\end{aligned}$$

i.e.

$$\begin{aligned}&(1-\pi_t)(R_tA_t^w+H_t^w+S_t^w-Z_t^w)\\&=\tfrac{\gamma_w\omega}{R_{t+1}\Omega_{t+1}}(R_{t+1}A_{t+1}^w+H_{t+1}^w+S_{t+1}^w-Z_{t+1}^w)+\frac{\gamma_w(1-\omega)\frac{1}{\gamma_r}(\varepsilon_{t+1})^{\frac{1}{1-\sigma}}\chi_r}{R_{t+1}\Omega_{t+1}}(R_{t+1}A_{t+1}^r+H_{t+1}^r+S_{t+1}^r-Z_{t+1}^r)].\end{aligned} \tag{C.2.30}$$

Using (C.2.20), (C.2.30) can be rewritten as

$$\begin{aligned}&(1-\pi_t)(R_tA_t^w+H_t^w+S_t^w-Z_t^w)\\&=\tfrac{1}{R_{t+1}}\tfrac{\gamma_w\omega}{\Omega_{t+1}}(R_{t+1}A_{t+1}^w+H_{t+1}^w+S_{t+1}^w-Z_{t+1}^w)+\tfrac{1}{R_{t+1}}(1-\tfrac{\gamma_w\omega}{\Omega_{t+1}})(R_{t+1}A_{t+1}^r+H_{t+1}^r+S_{t+1}^r-Z_{t+1}^r)].\end{aligned} \tag{C.2.31}$$

It is also possible to express the LHS of (C.2.30) as following:

$$(1-\pi_t)(R_t A_t^w + H_t^w + S_t^w - Z_t^w)$$
$$= (R_t A_t^w + W_t l_t^w - C_t^w - I_t^w p_t) + (H_t^w - W_t l_t^w) + (S_t^w - 0) - (Z_t^w - I_t^w p_t). \qquad \text{(C.2.32)}$$

Then, (C.2.33) holds and

$$(R_t A_t^w + W_t l_t^w - C_t^w - I_t^w p_t) + (H_t^w - W_t l_t^w) + (S_t^w - 0) - (Z_t^w - I_t^w p_t)$$
$$= \tfrac{1}{R_{t+1}} \tfrac{\gamma_w \omega}{\Omega_{t+1}} (R_{t+1} A_{t+1}^w + H_{t+1}^w + S_{t+1}^w - Z_{t+1}^w) + \tfrac{1}{R_{t+1}} (1 - \tfrac{\gamma_w \omega}{\Omega_{t+1}})(R_{t+1} A_{t+1}^r + H_{t+1}^r + S_{t+1}^r - Z_{t+1}^r)].$$

(C.2.33)

which implies that the worker's wealth at the end of time t after cash flows have occurred becomes either the worker's wealth or the retiree's wealth at the end of time t +1 and that the discounted risk−adjusted work probability

$$\tfrac{1}{R_{t+1}} \tfrac{\gamma_w \omega}{\Omega_{t+1}} \qquad \text{(C.2.34)}$$

will play a role in the valuation of worker's human wealth(nonfinancial asset) i.e., $H_t^w + S_t^w - Z_t^w$, and the discounted risk−adjusted retirement transition probability

$$\tfrac{1}{R_{t+1}} (1 - \tfrac{\gamma_w \omega}{\Omega_{t+1}}) \qquad \text{(C.2.35)}$$

will be used when the worker goes from work at time t to retirement at time t+1.

It is obvious that from (C.2.2) and $A_{t+1}^w = A_{t+1}^r$,

$$R_t A_t^w + W_t l_t^w - C_t^w - I_t^w p_t = \tfrac{\gamma_w \omega}{\Omega_{t+1}} A_{t+1}^w + (1 - \tfrac{\gamma_w \omega}{\Omega_{t+1}}) A_{t+1}^r. \qquad \text{(C.2.36)}$$

Thus, we obtain

$$\tfrac{1}{R_{t+1}} \tfrac{\gamma_w \omega}{\Omega_{t+1}} (H_{t+1}^w + S_{t+1}^w - Z_{t+1}^w) + \tfrac{1}{R_{t+1}} (1 - \tfrac{\gamma_w \omega}{\Omega_{t+1}}) \gamma_r (H_{t+1}^r + S_{t+1}^r - Z_{t+1}^r) = (H_t^w + S_t^w - Z_t^w) - W_t l_t^w + I_t^w p_t$$

(C.2.37)

It is not necessary that the following equations

$$H_t^w = W_t l_t^w + \tfrac{1}{R_{t+1}} \tfrac{\gamma_w \omega}{\Omega_{t+1}} H_{t+1}^w + \tfrac{1}{R_{t+1}} (1 - \tfrac{\gamma_w \omega}{\Omega_{t+1}}) \gamma_r H_{t+1}^r , \tag{C.2.38}$$

$$S_t^w = \tfrac{1}{R_{t+1}} \tfrac{\gamma_w \omega}{\Omega_{t+1}} S_{t+1}^w + \tfrac{1}{R_{t+1}} (1 - \tfrac{\gamma_w \omega}{\Omega_{t+1}}) \gamma_r S_{t+1}^r , \tag{C.2.39}$$

$$Z_t^w = I_t^w p_t + \tfrac{1}{R_{t+1}} \tfrac{\gamma_w \omega}{\Omega_{t+1}} Z_{t+1}^w + \tfrac{1}{R_{t+1}} (1 - \tfrac{\gamma_w \omega}{\Omega_{t+1}}) \gamma_r Z_{t+1}^r . \tag{C.2.40}$$

hold true. Eq. (C.2.37) to derive aggregate demand, supply and steady state endogenous variables. But it is convenient to use equations (C.2.38), (C.2.39) and (C.2.40) for calculation.

To confirm a solution for the value function, conjecture that

$$V_t^w = \Delta_t^w (C_t^w)^\upsilon (I_t^w)^\nu (1 - l_t^w)^{1-\upsilon-\nu} = \Delta_t^w (C_t^w)^\upsilon (\tfrac{\nu}{\upsilon p_t})^\nu (\tfrac{1-\upsilon-\nu}{\upsilon W_t})^{(1-\upsilon-\nu)} . \tag{C.2.41}$$

Then, to obtain an expression for Δ_t^w, substitute the conjectured solution for V_t^w into the objective to obtain

$$\Delta_t^w C_t^w (\tfrac{\nu}{\upsilon p_t})^\nu (\tfrac{1-\upsilon-\nu}{\upsilon W_t})^{(1-\upsilon-\nu)}$$
$$= \left[\left\{ C_t^w (\tfrac{\nu}{\upsilon p_t})^\nu (\tfrac{1-\upsilon-\nu}{\upsilon W_t})^{(1-\upsilon-\nu)} \right\}^\rho + \beta \gamma_w \left\{ \omega \Delta_{t+1}^w C_{t+1}^w (\tfrac{\nu}{\upsilon p_{t+1}})^\nu (\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}})^{(1-\upsilon-\nu)} + (1-\omega) \Delta_{t+1}^r C_{t+1}^r (\tfrac{\nu}{\upsilon p_{t+1}^r})^\nu (\tfrac{1-\upsilon-\nu}{\upsilon W_{t+1}^r})^{(1-\upsilon-\nu)} \right\}^\rho \right]^{1/\rho} .$$

(C.2.42)

Here, placing (C.2.21) into (C.2.42), (C.2.42) can be rewritten as

$$\left\{ \Delta_t^w C_t^w (\tfrac{\nu}{\upsilon p_t})^\nu (\tfrac{1-\upsilon-\nu}{\upsilon W_t})^{(1-\upsilon-\nu)} \right\}^\rho$$
$$= \left\{ C_t^w (\tfrac{\nu}{\upsilon p_t})^\nu (\tfrac{1-\upsilon-\nu}{\upsilon W_t})^{(1-\upsilon-\nu)} \right\}^\rho + \beta \gamma_w \left[\Delta_{t+1}^w \left\{ (\tfrac{W_t}{W_{t+1}})^{(1-\upsilon-\nu)\rho} (\tfrac{p_t}{p_{t+1}})^{\nu\rho} R_{t+1} \Omega_{t+1} \beta \right\}^\sigma C_t^w (\tfrac{\nu}{\upsilon p_t})^\nu (\tfrac{1-\upsilon-\nu}{\upsilon W_t})^{(1-\upsilon-\nu)} \right]^\rho .$$

(C.2.43)

Thus, applying that $\sigma\rho + 1 = \sigma$ and $\sigma\rho = \sigma - 1$, we have

$$(\Delta_t^w)^\rho = 1 + [(\frac{W_t}{W_{t+1}})^{(1-\upsilon-\nu)}(\frac{p_t}{p_{t+1}})^\nu R_{t+1}\Omega_{t+1}]^{\sigma-1}\beta^\sigma\gamma_w(\Delta_{t+1}^w)^\rho . \quad \text{(C.2.44)}$$

Now, checking that (C.2.44) is identical to (C.2.26), we obtain

$$\Delta_t^w = (\pi_t)^{-\frac{1}{\rho}} . \quad \text{(C.2.45)}$$

Thus,

$$V_t^w = (\pi_t)^{-\frac{1}{\rho}} C_t^w (\frac{\nu}{\upsilon p_t})^\nu (\frac{1-\upsilon-\nu}{\upsilon W_t})^{(1-\upsilon-\nu)} . \quad \text{(C.2.46)}$$

논문요약

보험, 사회보장과 경제

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기존의 연구들은 중첩세대모형에서 보험을 고려하지 않았던 반면, 본 논문에서는 중첩세대모형에 보험을 도입하고, 보험이 경제 주체들의 행동 및 거시경제에 미치는 영향을 분석한다. 본 연구는 Gertler의 1999년 논문에 제시된 중첩세대모형을 바탕으로 하며, Gertler 모형의 재유도 및 수정, 보험 모형의 개발 등 세 가지 주제에 초점을 두고 있다. 본 연구의 기여점은 다음과 같다.

첫째, 탄력적 노동 공급을 전제로 할 때 경제주체들의 의사결정 문제를 공식의 재유도를 통해 명확히 하였다. 그리고 한국 경제에 이 모형을 적용하여 사회보장 강화 및 고령화 현상 등이 세대 간의 자산 배분 및 노동 공급과 소비에 미치는 영향을 분석하였다.

둘째, 근로자의 의사결정 문제와 관련된 위험 조정 요인의 오류를 수정하고 비금융자산 간 가치평가 방법의 일관성이 유지되도록 하였다. Gertler 모형에서는 위험조정요인에 은퇴자의 생존확률이 고려되지 않았고, 근로자의 사회보장 자산 평가 시 불명확한 전환 인자가 포함되어 있어 사회보장자산의 가치를 과대평가하는

경향이 있었다. 따라서 이러한 문제점을 수정하여 개선된 모형을 제시한다.

셋째, Gertler의 수정된 중첩세대모형에 보험 요소를 도입하여 보험 모형을 제시한다. 보험 수요는 효용을 극대화하는 개인 의사결정에 의해 결정되며, 보험 구입은 개인의 효용을 직접적으로 증가시킨다. 하지만, 민영보험에 의해 완전히 보장되지 못한 경제적 손실은 효용에 직접 반영되지 않는 한계점이 있다. 따라서 GDP 대비 총손실의 비율이 일정하다는 가정 하에 민영보험에 의해 보장되지 않는 손실을 정부가 보장해주는 사회보험을 도입하여 이러한 한계를 보완하였다. 즉, 민영보험과 사회보험이 손실에 대한 보상을 제공하는 역할을 분담하는 틀을 마련하였다. 보험 모형은 기존의 경제 모형에서는 인식하지 못했던 손실을 인식할 수 있고, 보험의 미시적, 거시적 경제 효과를 분석하는 것이 가능하다.

마지막으로 보험 모형에 생명보험을 추가하여 확장된 형태의 보험 모형을 제시한다. 생명보험의 도입으로 근로자와 은퇴자 외에 피부양자에게 경제주체의 역할이 부여되고, 근로자의 사망 가능성의 도입으로 이전 확률이 변화하면서 근로자의 상태 변화가 다양해지고, 의사 결정에 사용되는 위험조정요인 및 가치평가요인이 변경된다. 생명보험 모형은 보다 현실적인 인구 구조를 반영하고 보험의 유형을 다양화하였다는 점에서 앞서 제시한 보험 모형과 차이가 있다.

주제어: 중첩세대모형, 보험, 사회보장, 자산가치평가, 정상상태